On Hyperdimensional Physics… and More….

With Comments by Dr. G J Munn, PhD. Physics

On Hyperdimensional Physics… and More….

Part I (**back**)

In a tragedy for science (if not for society in general) whose outlines we are only now beginning to appreciate, after Maxwell's death, two other 19th Century "mathematical physicists" -- **Oliver Heaviside and William Gibbs** -- "streamlined" Maxwell's original equations down to *four* simple (if woefully incomplete!) expressions. Because Heaviside openly felt the quaternions were "an abomination" -- never fully understanding the linkage between the critical scalar and vector components in Maxwell's use of them to describe the potentials of empty space ("apples and oranges," he termed them) -- Heaviside eliminated over *200 quaternions* from Maxwell's original theory in his attempted "simplification."

Oliver Heaviside, described by Scientific American (Sept. 1950) as "self-taught and ... never connected with any university ... had [however] a remarkable and inexplicable ability (which was possessed also by Newton and Laplace ...) to arrive at mathematical results of considerable complexity without going through any conscious process of proof ..." According to other observers, Heaviside actually felt that Maxwell's use of quaternions and their description of the "potentials" of space was "... *mystical, and should be murdered from the theory ...*" which -- by drastically editing Maxwell's original work after the latter's untimely death (from cancer), excising the scalar component of the quaternions and eliminating the hyperspatial characteristics of the directional (vector) components – Oliver Heaviside effectively accomplished singlehanded.

This means, of course, that the four surviving "classic" **Maxwell's Equations** -- which appear in every electrical and physics text the world over, as *THE* underpinnings of *ALL* 20th Century electrical and electromagnetic engineering, from radio to radar, from television to computer science, if not inclusive of every "hard" science from physics to chemistry to astrophysics that deals with electromagnetic radiative processes -- never appeared in *any* original Maxwell' paper or treatise! They are, in fact--

"Heaviside's equations!"

Lest anyone doubt this is the case, they merely have to read a highly revealing paper on the subject by another renowned British mathematical physicist of ***this*** century, Sir Edmund Whittaker, titled simply "Oliver Heaviside" (*Bulletin of the Calcutta Mathematical Society*, Vol. 20, 1928-29, p.202); or, another overview of Heaviside by Paul J. Nahin, "Oliver Heaviside: Sage in Solitude" (IEEE Press, New York, 1988, p.9, note 3.).

The end result was that physics lost its promising theoretical beginnings to becoming truly "hyperdimensional" physics *... over a century ago ...* and all that that implies.

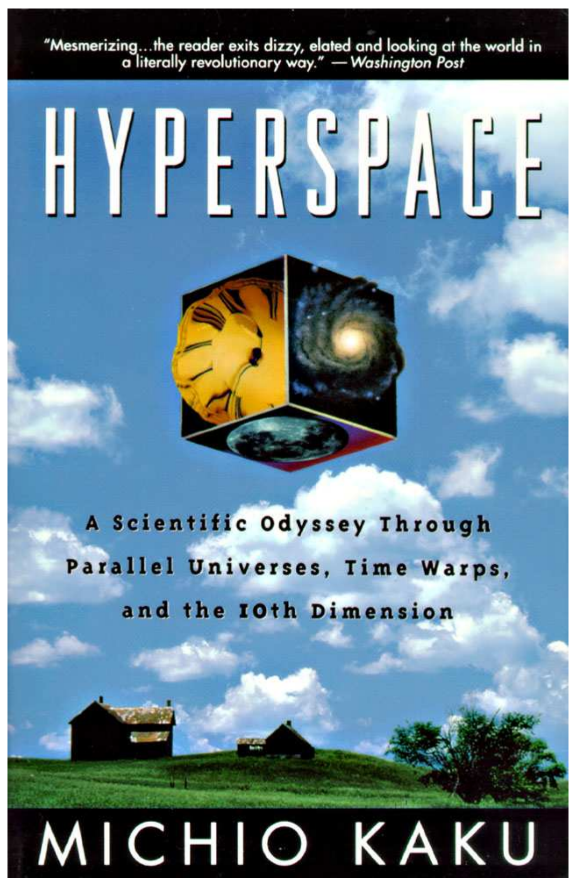
Georg Bernard Riemann mathematically initiated the 19th Century scientific community (if not the rest of Victorian society) into the "unsettling" idea of "hyperspace," on June 10, 1854. In a seminal presentation made at the University of Gottinggen in Germany, Riemann put forth the first mathematical description of the possibility of "higher, unseen dimensions ..." under the deceptively simple title: "On the Hypotheses Which Lie at the Foundation of Geometry."

Riemann's paper was a fundamental assault on the 2000-year old assumptions of "Euclidian Geometry" -- the ordered, rectilinear laws of "ordinary" three-dimensional reality. In its place, Riemann proposed a *four-dimensional* reality (of which our 3-D reality was merely a "subset"), in which the geometric rules were radically different, but also internally self-consistent. Even more radical: Riemann proposed that the basic laws of nature in 3-space, the three mysterious forces then known to physics -- electrostatics, magnetism and gravity -- were all fundamentally *united* in 4-space, and merely "looked different" because of the resulting "crumpled geometry" of our three-dimensional reality...

In terms of actual physics, Riemann was suggesting something clearly revolutionary: a major break with Newton's "force creates action-at-a-distance" theories of the time, which had been proposed to explain the "magical" properties of magnetic and electrical attraction and repulsion, gravitationally-curved motions of planets ... and falling apples, for over 200 years; in place of Newton, Riemann was proposing that such "apparent forces'" are a direct result of objects moving through 3-space "geometry" *... distorted* by the intruding geometry of "4-space!"

It is clear that Maxwell and other "giants" of 19th Century physics (Kelvin, for one), as well as an entire contemporary generation of 19th Century mathematicians (like Cayle, Tait, etc.) took Riemann's ideas very much to heart; Maxwell's original selection of *4-space quaternions* as the mathematical operators for his force equations and descriptions of electrical and magnetic interaction, clearly demonstrate his belief in Riemann's approach; and, his surprising literary excursions into poetry -- vividly extolling the implications of "higher-dimensional realities" ... including musings on their relationship to the ultimate origin of the *human soul* (above) -- emphatically confirm this outlook.

So, how can modern "hyperdimensional physicists" -- like **Michio Kaku**, at City College of the City University of New York-- representative of an entirely new generation of physical scientists now reexamining these century-old implications of "hyperspatial geometries" for generating the basic laws of Reality itself, almost casually claim:



> "... In retrospect, Riemann's famous lecture was popularized to a wide audience via *mystics, philosophers and artists, but did little to further our understanding of nature ...* First, there was no attempt to use hyperspace to simplify the laws of nature. Without Riemann's original guiding principle -- that the laws of nature become simple in higher dimensions -- scientists during this period were groping in the dark. Riemann's seminal idea of using geometry -- that is, crumpled hyperspace -- to explain the essence of a a force' was forgotten during those years ... The mathematical apparatus developed by Riemann became a province of pure mathematics, contrary to Riemann's original intentions. *Without field theory,* you cannot make any predictions with hyperspace [emphasis added]..."
>
> -- M. Kaku, "Hyperspace"
> [Doubleday (Anchor Books): New York, 1994]

Kaku's statement belies the entire "modern" outlook on 19th Century physics, and leaves the distinct impression of an apparently unconscious "bias" similar to Heaviside's, regarding Maxwell's actual treatment of such matters; certainly, in completely ignoring Maxwell's true discussion of the importance of the underlying *four-dimensional* "scalar potentials" for creating such "fields." And remember: Heaviside also thought of such "potentials" as ... "mystical ..."

The use of little-known **Hamiltonian 4-space quaternions**, to represent the effect of "scalar potentials" on electric charges (as opposed to Heaviside's *vectorial* descriptions of direct "electric force fields") obviously have led to great confusion; because ... Maxwell's "scalar potentials" are, of course, nothing short of exactly what Riemann initially proposed--

Quantifiable "geometric spatial distortions" ... the exact marriage of hyperspatial geometry and field theory that Kaku and others mistakenly believe (because they're basing their analysis on Heaviside's surviving vectorial version of Maxwell's original "Equations") is totally *missing* from this greatest achievement of 19th Century physics!

The major source of confusion surrounding Maxwell's actual Theory, versus what Heaviside reduced it to, is its math -- a notation system perhaps best described by H.J. Josephs ("The Heaviside Papers found at Paignton in 1957," *Electromagnetic Theory by Oliver Heaviside*, Including an account of Heaviside's unpublished notes for a fourth volume, and with a forward by Sir Edmund Whittaker, Vol. III, Third Edition, Chelsea Publishing Co., New York, 1971).

According to Josephs:

> "**Hamilton's algebra of quaternions**, unlike Heaviside's algebra of vectors, is not a mere abbreviated mode of expressing Cartesian analysis, *but is an independent branch of mathematics with its own rules of operation and its own special theorems. A quaternion is, in fact, a generalized or hypercomplex number ...* [emphasis added]"

And, you will remember, in 1897 **Hathaway** published a paper specifically identifying these hypercomplex numbers as "... numbers in *four-dimensional space*" (above). Thus, modern physics' apparent ignorance of Maxwell's 19th Century success -- a mathematically-based, *four-dimensional* "field-theory" -- would seem to originate from a basic lack of knowledge of the true nature of Hamilton's quaternion algebra itself!

> [Apparently, unless a "hyperdimensional theory" is narrowly expressed in terms of a separate technique Riemann himself invented for his own N-dimensional mapping -- the so-called "metric tensor" -- modern physicists don't seem to be able to recognize it as a valid higher-dimensional model ... not even when it was written in its own, specifically-designed, *four-dimensional* mathematical notation! (Riemann's "metric tensor," BTW, is essentially a graphical checkerboard composed, for a 4-space description, of 16 numbers defining, for instance, field strength at each point in that four-dimensional space. It is NOT written in quaternions.)
>
> And, unless you track down an original 1873 copy of Maxwell's "*Treatise*," there is no easy way to verify the existence of Maxwell's "hyperdimensional" quaternion notation; for, by 1892, the Third Edition incorporated a "correction" to Maxwell's original use of "scalar potentials" (contributed by George Francis Fitzgerald -- whom Heaviside heavily admired) -- thus removing a crucial distinction between 4-space "geometric potential," and a 3-space "vector field," from all subsequent "Maxwellian theory." Which is why Kaku apparently doesn't realize that Maxwell's original equations *were*, in fact, the first *geometric 4-space field theory ...* expressed in specific 4-space terms ... the language of quaternions!

Just another measure of Heaviside's effectiveness ...]

One of the difficulties of proposing a "higher dimension" is that, inevitably, people (and scientists are people!), will ask: "Ok, where is it? Where is the fourth dimension' ..?"

One of the most persistent objections to the 4-space geometries of Riemann, Cayley, Tait ... and Maxwell, was that no experimental proof of a "fourth dimension" was readily apparent; one of the more easily understandable aspects of "higher dimensionality" was that, a being from a "lower dimension" (a two-dimensional "Flatlander," for instance) entering our "higher" three-dimensional reality, would appear to vanish instantly from the lower-dimensional world (and, consequently, appear just as suddenly in the higher dimension -- but geometrically distorted.) When he returned to his own dimension, he would just as "magically" reappear ...

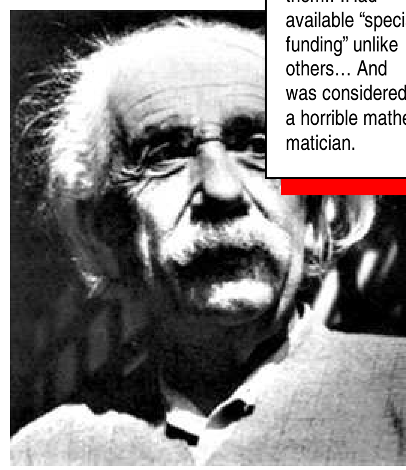
NONSENSE ! Secret Gov't has had time machines for 50+ years

Unfortunately (or fortunately, depending on your perspective ...) to the scientific mind, people in our dimension don't just "turn a corner one day ... and promptly vanish into Riemann's fourth dimension.'" While mathematically derivable and beautifully consistent, to "experimentalists" (and all real science ultimately has to be based on verifiable, independently repeatable experiments) there seemed no testable, physical *proof* of "hyperdimensional physics."

Einstein was a **INTELLECTUAL THIEF**, who STOLE most of his ideas and concepts from others (ie. Bernhard Reimann) and just reformulated them.. .Had available "special funding" unlike others... And was considered a horrible mathematician.

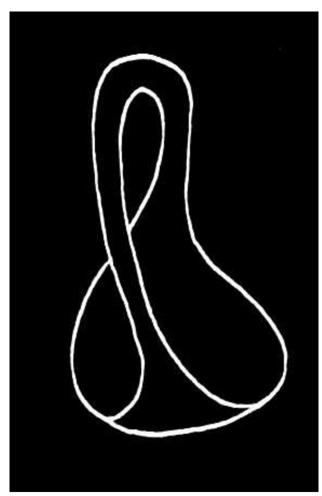
Thus "hyperspace"-- as a potential solution to unifying the major laws of physics -- after Maxwell's death, and the major rewriting of his Theory, quietly disappeared ... not to resurface for almost *half a century ...*

….until April of 1919.



At that time, a remarkable letter was delivered to one "**Albert Einstein**." Written by an obscure mathematician at the University of Konigsberg in Germany, **Theodr Kaluza**, the letter's first few lines offered a startling solution (at least, to Einstein -- unknowing of Maxwell's original quaternion equations) to one of physics' still most intractable problems: the mathematical unification of his own theory of gravity with Maxwell's theory of electromagnetic radiation ... via introduction of a *fifth* dimension. (Because Einstein, in formulating the General and Special Theory of Relativity in the intervening years since Riemann, had already appropriated *time* as the "fourth dimension," Kaluza was forced to specify his additional spatial dimension as "the fifth." In fact, this was the *same spatial dimension* as the 4-space designations used by Maxwell and his colleagues in their models ... *over 50 years before*.)

Despite its stunning (Einstein mulled over the paper's implications for more than two years, before finally supporting its scientific publication) mathematical success, in apparently -- finally -- uniting "gravity" and "light," the same question, "OK, where is it?" was asked of Kaluza as had been asked of Riemann, over 60 years before; because, there was no overt experimental proof (for instance, people and things up and "disappearing") of the physical existence of another spatial dimension. To which Kaluza this time had a very clever answer: he proposed that this "fourth dimension" -- unlike the other three we are familiar with -- somehow had collapsed down to a tiny circle ... "smaller than the smallest atom ..."

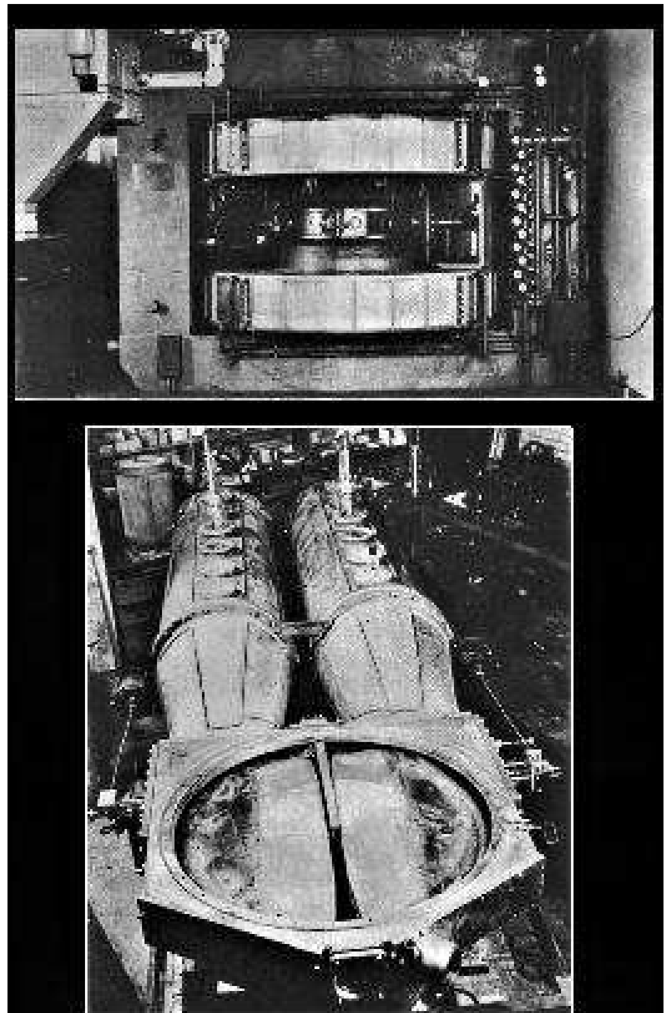


In 1926, another essentially unknown mathematician, **Oskar Klein**, was investigating the peculiar implications of Kaluza's ideas in the context of the newly invented atomic theory of "quantum mechanics." [Klein was a specialist in the truly arcane field of mathematical topology -- the *higher dimensional surfaces* of objects; the twisted 3-D topology of the 2-D surface of a "Klein Bottle" is named specifically in his honor]. Quantum mechanics had just been proposed a year or so before Klein's further topological investigation of Kaluza's ideas, by Max Planck and many others rebelling against perceived limitations of Maxwell's (remember, heavily sanitized by Gibbs and Heaviside) classical Electromagnetic Theory. The "quantum mechanics " theory would eventually become a highly successful (if bizarre, by common-sense standards) non-geometric effort to describe interactions between "fundamental particles," exchanging "forces" through discrete "quantitized" particles and energy in the sub-atomic world. Eventually, combining the two inquiries, Klein theorized that, if it truly existed, Kaluza's new dimension likely had somehow collapsed down to the "Planck length" itself -- supposedly the smallest possible size allowed by these fundamental interactions. However, that size was only about ... 10-33 cm long!

Thus, the main obstacle to experimental verification of the Kaluza-Klein Theory (and the reason why people simply didn't "walk into the fourth dimension"), was that quantum mechanics calculations affirmed that the *only way* to physically probe such an infinitesimally tiny dimension was with a new machine ... an "atom smasher." There was only one small "technical" problem ...

The energy required would *exceed the output of all the power plants on Earth ...* and then some!

Thus, the brief "blip" of new interest in "hyperdimensional physics" -- the discussions of Kaluza-Klein among physicists and topologists -- "dropped through the floor" by the 1930's. This occurred both because of Klein's "proof" of the apparent *impossibility* of any direct experimental verification of additional dimensions ... and because of the dramatic revolution then sweeping the increasingly technological world of Big Science--



The M.I.T. cyclotron.

The flood of "verifications"gushing forth from atom smashers all around the world, feverishly engaged in probing the new area the experimentalists apparently *could* verify: the multiplying populations of "fundamental particles"spawned by the bizarre mathematical world (even more bizarre than "N-dimensions") of Quantum Mechanics.

30 more years would pass ... before (almost by mathematical "accident") in 1968, the current mainstream "flap" of renewed scientific interest in "hyperspace" would be, like the legendary Phoenix, "magically" reborn -- a theory now known as "Superstrings" ... in which fundamental particles, and "fields," are viewed as hyperspace vibrations of infinitesimal, *multi-dimensional strings ...* From those relatively inauspicious beginnings, stretching across more than 60 years, the current focus of scientific research papers on "hyperspace" -- from continued research into updated versions of the old "Kaluza-Klein Theory"; to discussions of the much newer "Supergravity" hyperspace unification model; to the exotic "String Theory" itself -- has grown geometrically (over 5000 papers by 1994 alone, according to Michio Kaku -- see above). This much attention to a subject involving realities you can't even *see*, represents nothing short of a fundamental psychological revolution sweeping across a major segment of the worldwide scientific community.

For most physicists currently interested in the problem, the "Superstring" hyper-dimensional model has overwhelming advantages over all its predecessors. Besides effectively unifying all the known forces of the Universe ... from electromagnetism to the nuclear force ... in a literally beautiful "ultimate"picture of Reality, it also makes a *specific prediction* about the total number of N-dimensions that can form:

"Ten" (or "26," depending on the rotation of the "strings").

The bad news is: *they can't be tested either ...*

As all *ten dimensions* are curled up (in the model) inside the same experimentally unreachable "Planck length" which spelled the scientific demise of the original Kaluza-Klein ...

Impasse.

This, then is the current situation.

The "hottest" mainstream scientific theory to come along in more than half a century, the next best thing to a "Theory of Everything" (and seriously attempting to become precisely *that* ...), is not only a Hyperdimensional Model of Reality ... it is *another* one which, by its fundamental nature--

Can't scientifically be tested!

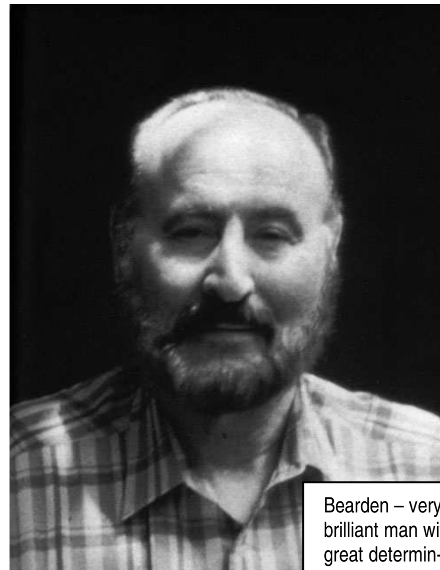
While a "hyperdimensional model" which can be tested easily -- as this paper will unequivocally show -- for over a 100 years has been systematically ignored.

Is it just us ... or is there something truly *wrong* with this picture?

On Hyperdimensional Physics… and More….

Part II (back)

Lt. Col Thomas E. Bearden, retired army officer and physicist, has been perhaps the most vocal recent proponent for restoring integrity to the scientific and historical record regarding James Clerk Maxwell -- by widely promulgating his original equations; in a series of meticulously documented papers on the subject, going back at least 20 years, Bearden has carried on a relentless one-man research effort regarding what Maxwell *really* claimed. His painstaking, literally thousands of man-hours of original source documentation has led directly to the following, startling conclusion:

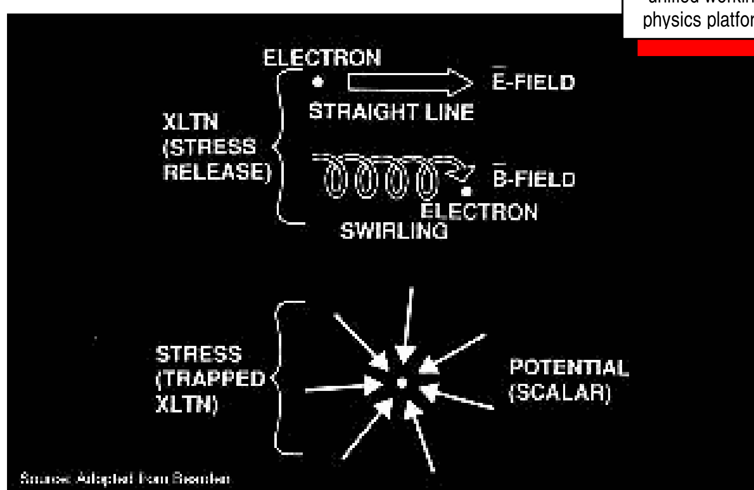


Bearden – very brilliant man with great determination. Ressurected previous scientific info - in conjunction with present day postulates – to form a more discernable and "unified working physics platform!

Maxwell's original theory is, in fact, the true, so-called "Holy Grail" of physics *... the first successful unified field theory in the history of Science ...* a fact apparently completely unknown to the current proponents of "Kaluza-Klein," "Supergravity," and "Superstring" ideas ....

Just *how* successful, Bearden documents below:

" ... In discarding the scalar component of the quaternion, Heaviside and Gibbs unwittingly discarded the unified EM/G [electromagnetic/ gravitational] portion of Maxwell's theory that arises when the translation/directional components of two interacting quaternions reduce to zero, but the scalar resultant remains and *infolds* a deterministic, dynamic structure that is a function of oppositive directional/translational components. In the infolding of EM energy inside a scalar potential, a *structured scalar* potential results, almost precisely as later shown by Whittaker but unnoticed by the scientific community. The simple vector equations produced by Heaviside and Gibbs captured only that subset of Maxwell's theory where EM and gravitation are mutually exclusive. In that subset, electromagnetic circuits and equipment will not ever, and *cannot* ever, produce gravitational or inertial effects in materials and equipment.



Two kinds of EM energy: Stress and translation. EM translation energy does external work. EM stress energy does internal work on the medium.

"Brutally, not a single one of those Heaviside/ Gibbs equations ever appeared in a paper or book by James Clerk Maxwell, even though the severely restricted Heaviside/Gibbs interpretation is universally and erroneously taught in all Western universities as *Maxwell's* theory.

"As a result of this artificial restriction of Maxwell's theory, Einstein also inadvertently restricted his theory of general relativity, forever preventing the unification of electromagnetics and relativity. He also essentially prevented the present restricted general relativity from ever becoming an experimental, engineerable science on the laboratory bench, since a hidden internalized electromagnetics causing a deterministically structured local space-time curvature was excluded.

"Quantum mechanics used only the Heaviside/ Gibbs *externalized* electromagnetics and completely missed Maxwell's *internalized and ordered* electromagnetics enfolded inside a structured scalar potential. Accordingly, QM [quantum mechanics] maintained its Gibbs statistics of quantum change, which is nonchaotic a priori. Quantum physicists by and large excluded Bohm's hidden variable theory, which conceivably could have offered the potential of *engineering quantum change -- engineering physical reality itself.*

"Each of these major scientific disciplines missed and excluded a subset of their disciplinary area, because they did not have the scalar component of the quaternion to incorporate. Further, they completely missed the significance of the Whittaker approach, which already shows how to apply and engineer the very subsets they had excluded.

"What now exists in these areas are three separate, inconsistent disciplines. Each of them unwittingly excluded a vital part of its discipline, *which was the unified field part*. Ironically, then, present physicists continue to exert great effort to find the missing key to unification of the three disciplines, but find it hopeless, because *these special subsets are already contradictory to one another,* as is quite well known to foundations physicists.

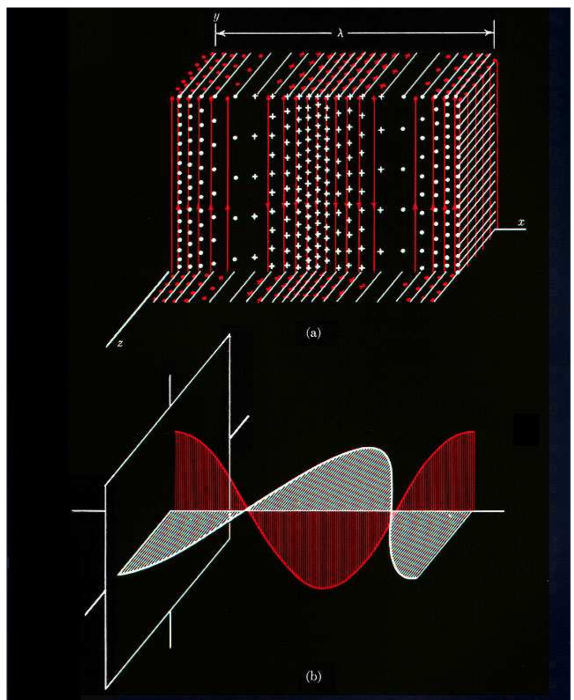
"Obviously, if one wishes to unify physics, one must add back the unintentionally excluded, unifying subsets to each discipline. Interestingly, all three needed subsets turn out to be one and the same ..."

-- T.E. Bearden, "Possible Whittaker Unification of
Electromagnetics, General Relativity, and Quantum Mechanics,"
(Association of Distinguished American Scientists
2311 Big Cove Road, Huntsville, Alabama, 35801)

Given Bearden's analysis -- what did we actually lose ... when science "inadvertently lost Maxwell ..?"

If two key physics papers often cited by Bearden (which appeared decades after the death of Maxwell), are accurate ... we lost nothing less than--

The "electrogravitic" control of gravity itself!!



(a) Electric and magnetic field distribution of a plane electromagnetic wave at any one instant. The electrostatic lines of force are drawn in red and the magnetic lines of force are drawn in black. (b) A conventionalized diagram of the wave form at the same instant. (Reprinted from *Ultra-High-Frequency Radio Engineering* by W. L. Emery, by permission of The Macmillan C

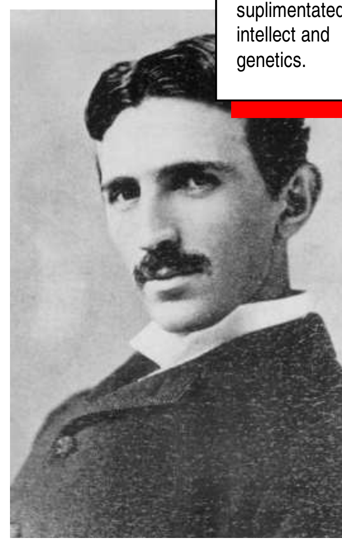
The critically important research cited by Bearden was originally published by "**Sir Edmund Whittaker**" (the same cited earlier in *this* paper), beginning in 1903. The first was titled "On the partial differential equations of mathematical physics" (*Mathematische Annalen*, Vol. 57, 1903, p.333-335); the second, "On an Expression of the Electromagnetic Field due to Electrons by means of two Scalar Potential Functions" (Proceedings of the London Mathematical Society, Vol.1, 1904, p. 367-372).

Whittaker, a leading world-class physicist himself, single-handedly rediscovered the "missing" scalar components of Maxwell's original quaternions, extending their (at the time) unseen implications for finally uniting "gravity" with the more obvious electrical and magnetic components known as "light." In the first paper, as Bearden described, Whittaker theoretically explored the existence of a "hidden" set of electromagnetic waves traveling in *two simultaneous directions* in the scalar potential of the vacuum -- demonstrating how to use them to curve the local and/or distant "space-time" with *electromagnetic radiation*, in a manner directly analogous to Einstein's later "mass-curves-space" equations. This key Whittaker paper thus lays the direct mathematical foundation for an *electrogravitic* theory/technology of gravity control. In the second paper, Whittaker demonstrated how two "Maxwellian scalar potentials of the vacuum" -- gravitationally curving space-time -- could be turned back into a detectable "ordinary" electromagnetic field by two interfering "scalar EM waves"... even at a distance.

Whittaker accomplished this by demonstrating mathematically that "the field of force due to a gravitating body can be analyzed, by a spectrum analysis' as it were, into an infinite number of constituent fields; and although the whole field of force does not vary with time, *yet each of the constituent fields is an undulatory character, consisting of a simple-disturbance propagated with uniform velocity ...* [and] the waves will be longitudinal (top) ... These results assimilate the propagation of gravity to that of light ... [and] would require that gravity be propagated with a finite velocity, which however *need not be the same as that of light* [emphasis added], and may be enormously greater ..." (Op. Cit., "On the partial differential equations of mathematical physics")

Sheer unmitigated "genius"! With the disctint ability to forsee the conclusion of his thoughts and concepts "**BEFORE**" commencing with them. By the way, he had help... extraterrestrial help.. .that is.. suplimentated intellect and genetics.

Remarkably, four years *before* Whittaker's theoretical analysis of these potentials (pun intended ...), on the evening of July 3-4, 1899, **Nikola Tesla** (right) -- the literal inventor of modern civilization (via the now worldwide technology of "alternating current") -- experimentally anticipated "Whittaker's interfering scalar waves" by finding them in nature; from massive experimental radio transmitters he had built on a mountain top in Colorado, he was broadcasting and receiving (by his own assertion) *"longitudinal stresses"* (as opposed to conventional EM "transverse waves") through the vacuum. This he was accomplishing with his own, hand-engineered equipment (produced according to Maxwell's original, quaternion equations), when he detected an interference "return" from a passing line of thunderstorms. Tesla termed the phenomenon a "standing columnar wave," and tracked it electromagnetically for hours as the cold front moved across the West (Nikola Tesla, Colorado Springs Notes 1899-1900, Nolit, Beograd, Yugoslavia, 1978 pp. 61-62).



[Many have since speculated that Tesla's many other astonishing (to the period) technological accomplishments, many of which apparently "were lost" with his death in 1942, were based on this true understanding of Maxwell's original, "hyperdimensional" electromagnetic ideas ...]

Tesla's experimental earlier detection notwithstanding, what Whittaker theoretically demonstrated years after Tesla was that future electrical engineers could also take Maxwell's original *4-space*, quaternion description of electromagnetic waves (the real "Maxwell's Equations"), add his own (Whittaker's) specific gravitational potential analysis (stemming from simply returning Maxwell's scalar quaternions in Heaviside's version of "Maxwell's Equations"...), and produce a workable "unified field theory" (if not technology!) of *gravity control ...*

Unless by now, in some government "black project," *they already have--*

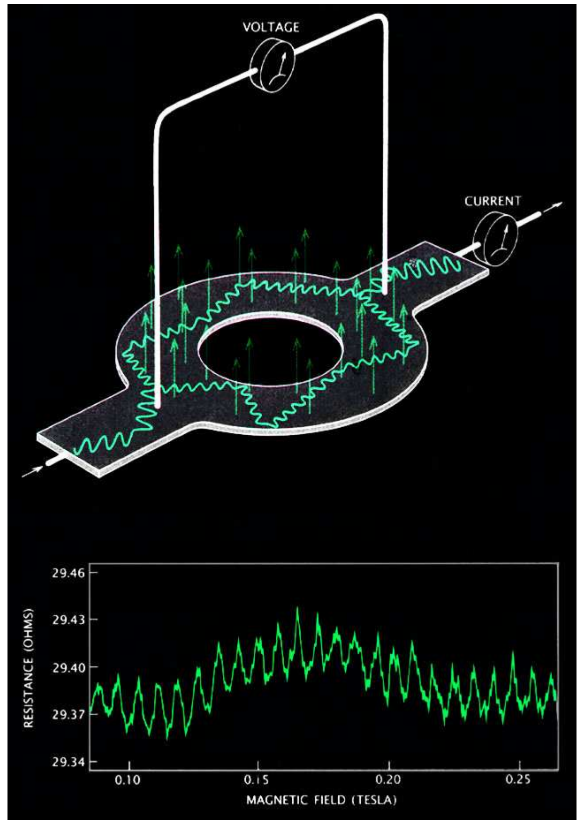
DUH!
Such sophisticated technology has been in use for 50 years under the guise of "black projects" that the poor bastard tax payer has been funding... and will ultimately be used against us.. under the NEW WORLD ORDER run by the godless satan-worshiping **ILLUMINATI**

And what we've deliberately been "leaked" over the last seven years, in repeated video images of "exotic vehicles" performing impossible, non-Newtonian maneuvers on official NASA TV shuttle coverage ... is simply the result!

Theory is one thing (Maxwell's or Whittaker's), but experimental results are supposedly the ultimate Arbiter of Scientific Truth. Which makes it all the more curious that Tesla's four-year *observational anticipation* of Whittaker's startling analysis of Maxwell -- the experimental confirmation of an electromagnetic "standing columnar (longitudinal) wave" in thunderstorms -- has been resolutely ignored by both physicists and electrical engineers for the past 100 years; as have the stunning NASA TV confirmations of "something" (above) maneuvering freely in Earth orbit.

With that as prologue, a new generation of physicists, also educated in the grand assumption that "Heaviside's Equations" are actually "Maxwell's," were abruptly brought up short in 1959 with another remarkable, equally elegant experiment -- which finally demonstrated in the laboratory the stark reality of Maxwell's "pesky scalar potentials" ... those same *"mystical"* potentials that Heaviside so effectively banished for all time from current (university-taught) EM theory.

In that year two physicists, **Yakir Aharonov** and **David Bohm**, conducted a seminal "electrodynamics" laboratory experiment ("Significance of Electromagnetic Potentials in Quantum Theory," *The Physical Review*, Vol. 115, No. 3, pp. 485-491; August, 1959). Aharonov and Bohm, almost 100 years after Maxwell first predicted their existence, succeeded in actually *measuring* the "hidden potential" of free space, lurking in Maxwell's original *scalar quaternion equations*. To do so, they had to cool the experiment to a mere 9 degrees above Absolute Zero, thus creating a total shielding around a *superconducting* magnetic ring



[for a slightly different version of this same experiment – (see diagram); the oscillation of electrical resistance in the ring (bottom graph) is due to the changing electron "wave functions" -- triggered by the "hidden Maxwell scalar potential" created by the shielded magnet -- see text, below].

Once having successfully accomplished this non-trivial laboratory set up, they promptly observed an "impossible" phenomenon:

Totally screened, by all measurements, from the *magnetic* influence of the ring itself, a test beam of electrons fired by Aharonov and Bohm at the superconducting "donut," nonetheless, *changed their electronic state* ("wave functions") as they passed through the observably "field-free" region of the hole -- indicating they were sensing "something," even though it could NOT be the ring's magnetic field. Confirmed now by decades of other physicists' experiments as a true phenomenon (and not merely improper shielding of the magnet), this "Aharonov-Bohm Effect" provides compelling proof of a deeper "spatial strain" -- a "scalar potential" -- underlying the existence of a so-called magnetic "force-field" itself. (Later experiments revealed a similar effect with shielded *electrostatic* fields ...)

All of which provides compelling proof of "something else," underlying all reality, capable of transmitting energy and information across space and time ... even in the *complete absence* of an electromagnetically detectable 3-D spatial "field"--

Maxwell's quaternion ... *hyperdimensional* "potential."

So, what does all this have to do with NASA's announcement of a "new planet?"

If a "potential" *without a field* can exist in space -- as Maxwell's quaternion analysis first asserted, and Aharonov-Bohm "only" a century later ultimately found -- then, as defined by Maxwell in his comparisons of the aether with certain properties of laboratory "solids," such a potential is equivalent to an unseen, *vorticular* (rotating) "stress" in space. Or, in Maxwell's own words (first written in 1873 ...):

> "There are physical quantities of another kind [in the aether] which are related to directions in space, but which are not vectors. Stresses and strains in solid bodies are examples, and so are some of the properties of bodies considered in the theory of elasticity and in the theory of double [rotated] refraction. Quantities of this class require for their definition nine [part of the "27-line"...] numerical specifications. They are expressed in the language of quaternions by linear and vector functions of a vector ..."

-- J.C. Maxwell, "A Treatise on Electricity and Magnetism,"
(Vol.1, 3rd Edition, New York, 1954)

And stresses, when they are relieved, must release energy into their surroundings ...

There is now much fevered discussion among physicists, (~100 years post-Maxwell) of the Quantum Electrodynamics Zero Point Energy (ZPE) of space -- or, "the energy of the vacuum"; to many familiar with the original works of Maxwell, Kelvin, etc al., this sounds an awful lot like the once-familiar "aether" ... merely updated and now passing under "an assumed name." Thus, creating -- then relieving -- a "stress" in Maxwell's vorticular aether is precisely equivalent to tapping the "energy of the vacuum" -- which, according to current "quantum mechanics' models," possesses a staggering amount of such energy per cubic inch of space. Even inefficiently releasing a tiny percentage of this "strain energy" into our three dimensions -- or, into a *body* existing in three-dimensional space -- could make it appear as if the energy was coming from *nowhere* ... "something from *nothing*." In other words, to an entire generation of students and astrophysicists woefully ignorant of Maxwell's *real* equations, such energy would appear as--

"Perpetual motion!"

Given the prodigious amount of "vacuum energy" calculated by *modern* physicists (trillions of atomic bomb equivalents per cubic centimeter ...), even a relatively minor but sudden release of such vast vacuum (aether) stress potential *inside a planet ...* could literally destroy it--

Finally answering the crucial astrophysical objection to the "exploded planet model" that Van Flandern has been encountering ...

"But Tom -- just how do you blow up' *an entire world?!"*

The answer is now obvious: via hyperdimensional "vacuum stress energy" ... ala Whittaker and Maxwell.

As we shall show, it is this "new" source of energy -- in a far more "controlled" context -- that seems also to be responsible now for not only the "anomalous infrared excesses" observed in the so-called "giant outer planets" of this solar system--

It is this same source of energy (in the Hyperdimensional Physics Model) that, according to our analysis, must now be primarily responsible for the radiated energies of *stars* ... including the *Sun itself.*

~~~~~~

Since, in three dimensions, *all* energy eventually "degrades" to random motions -- via Kelvin and Gibb's 19th Century Laws of Thermodynamics (it's called "increasing entropy") -- "stress energy" of the aether (vacuum) released inside a material object, even if it initially appears in a coherent form -- driving, for instance, the anomalous (1400 mile-per-hour!), planet-girdling winds of distant Neptune's "jet streams" -- will eventually degrade to simple, random heat ... ultimately radiated away as "excess infrared emissions" into space. It's the initial, astrophysical conditions under which such "Maxwellian space potentials" can be released inside a planet (or a star ...), that have been the central focus of our efforts for ten years --

To create a predictive, mathematical "hyperdimensional model" of such physics.

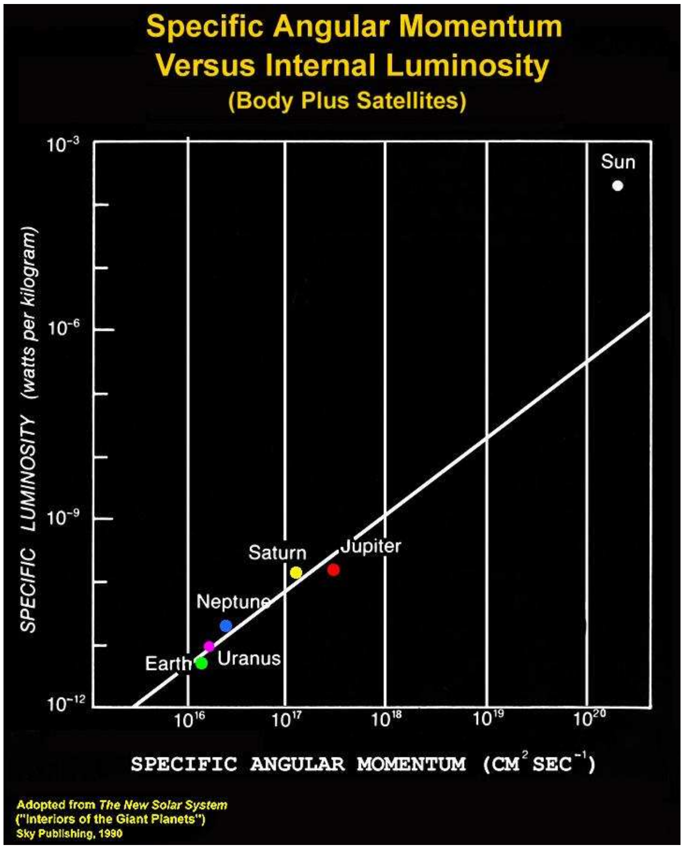
The entire question comes down to--
~~~~~~

"What set of known spatial conditions will slowly, predictably, release the potential strains of 4-space into 3-space' ... *inside* a massive world ... so that when this energy inevitably degrades to heat, its radiative signature identifies the original hyperdimensional' source?"

Fortunately, we are surrounded by almost half a dozen examples close at hand: the giant, "anomalously radiating" planets of *this* solar system (and some major moons). Over the past decade, as we have attempted to understand their anomalous IR radiation, one thing has become clear -- to a first order, the "infrared excesses" of the giant planets all seem to correlate very nicely with *one parameter* each has in common -- regardless of their individual masses, elemental compositions, or distance from the Sun:

Their total system "angular momentum."

The mass of a body and the rate at which it spins, in classical physics, determines an object's "angular momentum." In our Hyperdimensional Model, its a bit more complicated -- because objects apparently separated by distance in this (3-space) dimension are in fact *connected* in a "higher" (4-space) dimension; so, in the HD model, one also adds in the orbital momentum of an object's gravitationally-tethered satellites -- moons in the case of planets; planets, in the case of the Sun, or companion stars in the case of other stars.



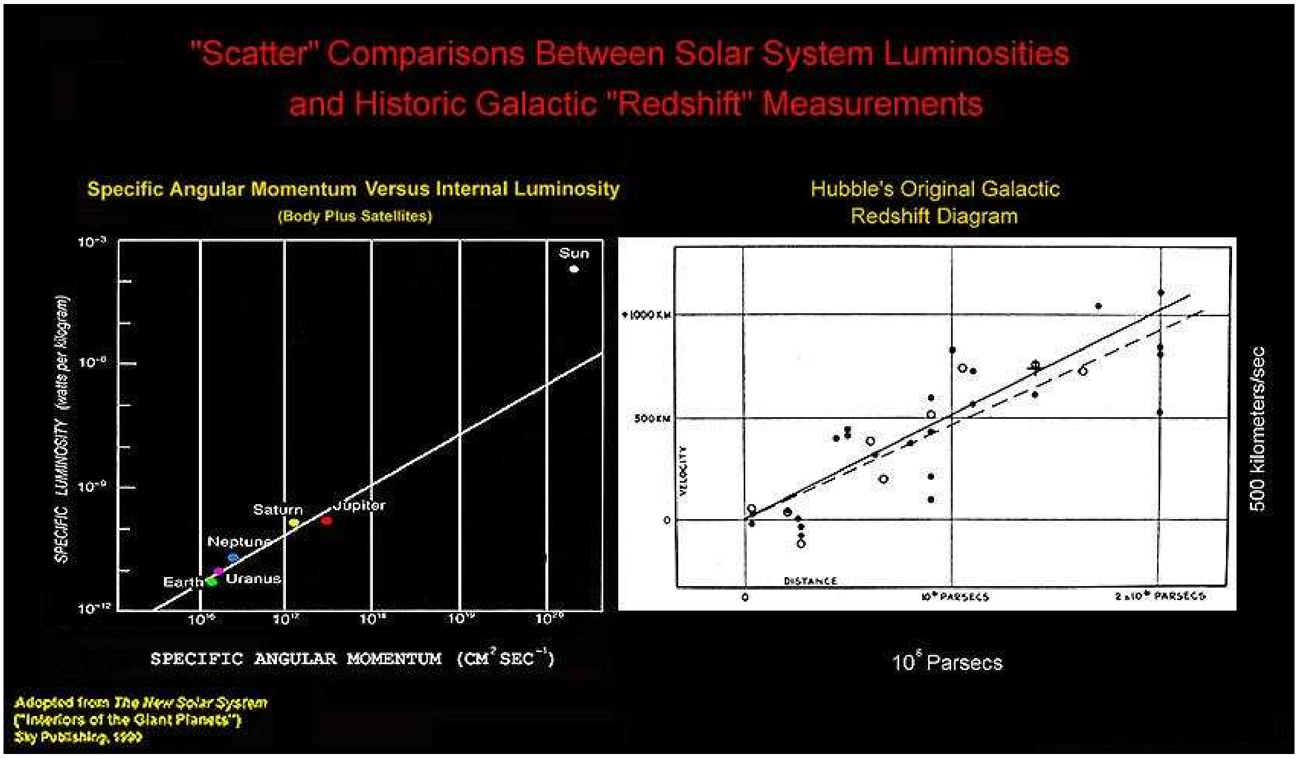
When one graphs the total angular momentum of a set of objects (see photo) such as radiating outer planets of this solar system (plus Earth and Sun) -- against the total amount of internal energy each object radiates to space, the results are striking:

The more *total* system angular momentum a planet (or any celestial body) possesses (as defined above -- object plus satellites), the greater its intrinsic "brightness," i.e. the more "anomalous energy" it apparently is capable of "generating."

And, as can be seen from this key diagram, this striking linear dependence now seems to hold across a range of luminosity and momentum totaling almost *three orders of magnitude* ... almost 1000/1!

Especially noteworthy, the Earth (not "a collapsing gas giant," by any stretch of the imagination) also seems to fit *precisely* this empirical energy relationship: when the angular momentum of the Moon is added to the "spin momentum" of its parent planet, the resulting correlation with measurements derived from internal "heat budget" studies of the Earth are perfectly fitted to this solar-system-wide empirical relationship -- even though the Earth's internal energy is *supposedly* derived from "radioactive sources."

And, as can be seen from the accompanying historical comparison, (see photo below) this striking solar system linear relationship is actually more tightly constrained (even at this early stage) than the original Hubble "redshift data" supporting the Big Bang!



This discovery contains major implications, not only for past geophysics and terrestrial evolution ... but for future geological and climatological events -- "Earth changes," as some have termed them. These may be driven, not by rising solar interactions or by-products of terrestrial civilization (accumulating "greenhouse gases" from burning fossil fuels), but by this same "hyperdimensional physics." If so, then learning a lot more about the mechanisms of this physics -- and quickly! -- is a critical step toward intervening and eventually controlling our future well being, if not our destiny, on (and off!) this planet ...

For the "Hyperdimensional Physics" model, this simple but powerful relationship now seems to be the equivalent of Relativity's $E=MC^2$: a celestial object's total internal luminosity seems dependent upon only one physical parameter:

L=mr2 = total system angular momentum (object, plus all satellites)

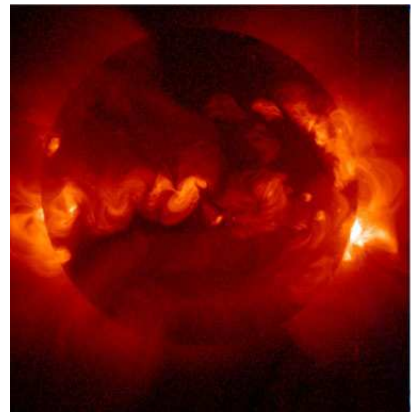
There is a well-known "rule of thumb" in science, perhaps best expressed by a late Noble Laureate, physicist Richard Feynman:

> "You can recognize truth by its beauty and simplicity. When you get it right, it is obvious that it is right -- at least if you have any experience -- because usually what happens is that more comes out than goes in ... The inexperienced, the crackpots, and people like that, make guesses that are simple, but you can immediately see that they are wrong, so that does not count. Others, the inexperienced students, make guesses that are very complicated, and it sort of looks as if it is all right, but I know it is not true because the truth always turns out to be simpler that you thought ..."

This startling relationship -- our discovery of the simple dependence of an object's internal luminosity on its total system angular momentum -- has that "feel" about it; it is simple ... it is elegant ... in fact--

It could even be true.

But, as can be seen from examining the luminosity/angular momentum diagram again, there also appears to be one glaring exception to this otherwise strikingly linear relationship:



The Sun itself.

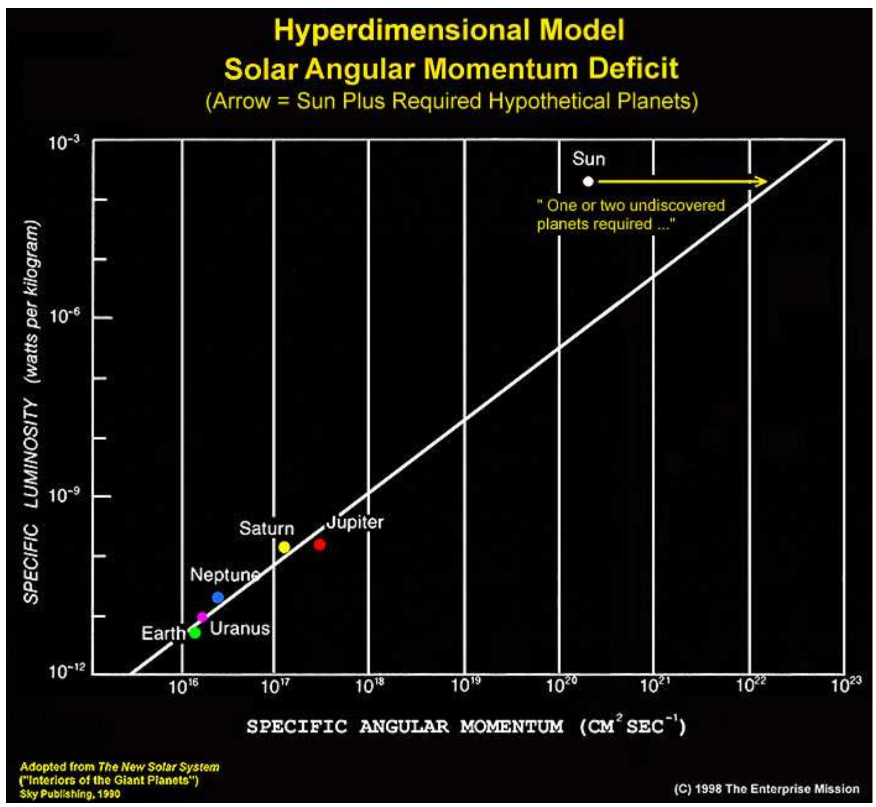
Independent research, involving over 30 years of attempted confirmation of the Sun's basic energy source -- in the form of solar/terrestrial observations of tiny atomic particles called "neutrinos," supposedly coming from the center of the Sun -- have left laboratory physicists and astrophysicists with a *major* astronomical enigma:

The Sun is not emitting *anything* like the number of neutrinos required by the "Standard Solar Model" for its observed energy emission; if its energy is due to "thermo-nuclear reactions" (as the Standard Model *demands*), then the observed "neutrino deficit" is upwards of 60%: even more remarkable, certain kinds of primary neutrinos (calculated as required to explain the bulk of the solar interior's fusion reactions, based on laboratory measurements) turn out to be simply *missing altogether!*

So -- what *really* fuels the Sun?

The answer to the Sun's apparent violation of the Standard Solar Model -- ironically, is contained in its striking "violation" of our key angular momentum/luminosity diagram:

In the Hyperdimensional Model, the Sun's primary energy source -- like the planets' -- must be driven by its total *angular momentum* -- its own "spin momentum," plus the total angular momentum of the planetary masses orbiting around it. Any standard astronomical text reveals that, though the Sun contains more than 98% of the mass of the solar system, it contains less than 2% of its total angular momentum. The rest is in the planets. Thus, in adding up their total contribution to the Sun's angular momentum budget -- if the HD model is correct -- we should see the Sun following the same line on the graph that the planets, from Earth to Neptune, do.



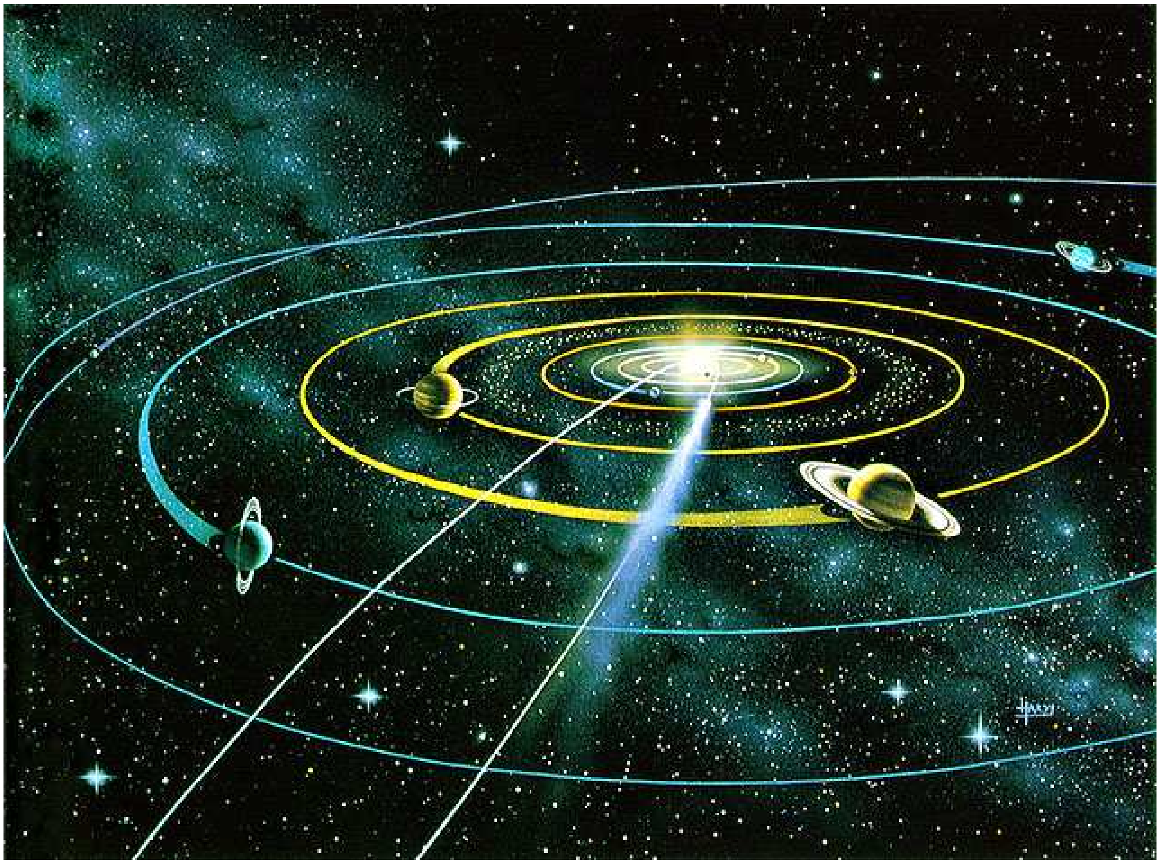
It doesn't.

The obvious answer to this dilemma is that the HD model is simply wrong.

The less obvious is that we're missing something ...

Like ... *additional planets* (above)!

By adding another big planet (or a couple of smaller ones) *beyond* Pluto (several hundred times the Earth's distance from the Sun -- below), we can move the Sun's total angular momentum to the right on the graph, until it almost intersects the line (allowing for a percentage, about 30%, of internal energy expected from genuine thermonuclear reactions ...). This creates the specific "HD prediction" that "the current textbook tally of the Sun's angular momentum is deficient because ..."



We haven't discovered all the remaining members of the solar our solar system yet!

YEA! YOU FOOLS… its called NIBIRU and our solar system's other dwarf sun and six sister satellite planets… that arrives **and destroys EVERY 3,600 years – LIKE CLOCKWORK!** It's at our doorstep right now preparing for huge destruction!! READ ALL ABOUT IT AT THIS WEBSITE.. under – **SECRETS!**

As a dividend, this promptly presents us with our <u>first key test of the Hyperdimensional Model</u>:

1) Find those planets!

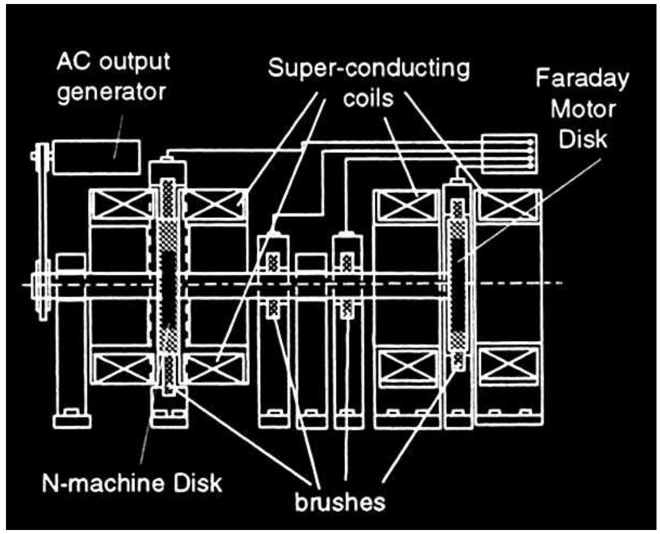
<u>The second test of the Hyperdimensional Model</u> is that, unlike other efforts to explain anomalous planetary energy emissions via continued "planetary collapse," or "stored primordial heat," the hyperdimensional approach specifically predicts one radical, *definitive* observational difference from all other existing explanations--

2) HD energy generation in both planets and stars should be -- must be -- *variable*.

This is simply implicit in the mechanism, which generates the hyperdimensional energy in the first place: ever *changing* hyperspatial geometry.

If the ultimate source of planetary (or stellar) energy is this "*vorticular* (rotating) spatial stress between dimensions" (ala Maxwell), then the constantly changing pattern (both gravitationally and dimensionally) of interacting satellites in orbit around a major planet/star must modulate that stress pattern as a constantly changing, *geometrically twisted* "aether" (ala Whittaker's amplifications of Maxwell). In our Hyperdimensional Model, it is this "constantly changing hyperspatial geometry" that is capable (via *resonant rotations* with the masses in question --

either as spin, or circular orbital motions) of extracting energy from this underlying "rotating, vorticular aether" ... and then releasing it *inside* material objects.



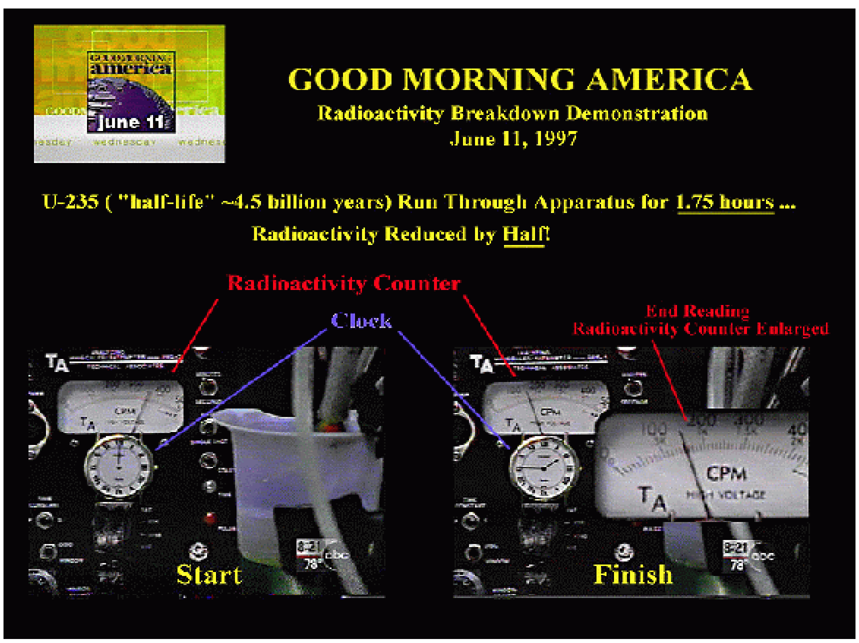
Initially, this "excess energy" can appear in many different forms -- high-speed winds, unusual electrical activity, even enhanced nuclear reactions -- but, ultimately, it must all degrade to simple "excess heat." Because of the basic physical requirement for *resonance* in effectively coupling a planet (or a star's) "rotating 3-D mass to the underlying 4-D aether rotation," this excess energy generation must also, inevitably, *vary* with time -- as the changing orbital geometry of the "satellites" interacts with the spinning primary (and the underlying, "vorticular aether"...) in *and out-of-phase.*

For these reasons, as stated earlier, *time-variability* of this continuing energy exchange must be a central hallmark of this entire "HD process."

[Incidentally, understanding this basic "hyperdimensional transfer mechanism," in terms of Maxwell's original quaternions (that describe "a rotating, vorticular, four-dimensional sponge-like aether"), immediately lends itself to creating a "Hyperdimensional *Technology* " based on this same mechanism.

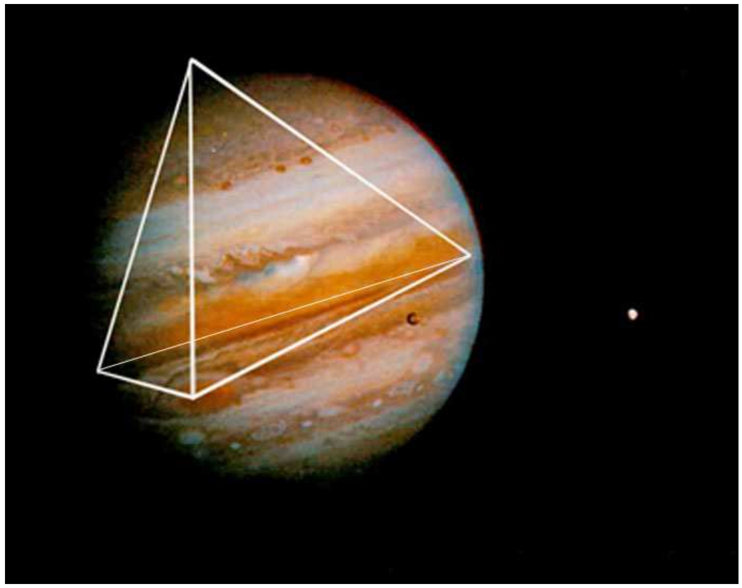
The fundamental "violations" of current physics exhibited by so-called "free energy" machines -- from the explicitly-rotating "N-machine" to the initially frustrating *time-variable aspects* of "electro-chemical cold fusion"-- are now elegantly explained by appropriate application f Maxwell's original ideas.
(see background articles of Tom Bearden)

Even more extraordinary: the recent startling demonstration, broadcast nationwide on ABC's "Good Morning America" last year, of a "physically impossible" major reduction -- in a *few minutes!* -- of long-lived radioactive Uranium isotopes. Normally, such processes require *billions of years* to accomplish. This too is now elegantly explained by the Hyperdimensional Model-- As -- an "induced hyperspatial stress," created by the machine ... the same stress that initially (in the Model) induces "unstable isotopes" in the first place. By technologically enhancing such vacuum stress within these nuclei, via a *retuning* of Maxwell's "scalar potentials," the normal radioactive breakdown process is accelerated -- literally *billions of times ...*



The implications for an entire "rapid, radioactive nuclear waste reduction technology" -- accomplishing in *hours* that would normally require *aeons* -- is merely one immediate, desperately needed world-wide application of such "Hyperdimensional Technologies."]

In our own planetary system, all the "giant" planets possess a retinue of at least a dozen satellites: one or two major ones (approximating the size of the planet Mercury) ... with several others ranging down below the diameter and mass of our own Moon ... in addition to a host of smaller objects; because of the "lever effect" in the angular momentum calculations, even a small satellite orbiting far away (or at a steep angle to the planet's plane of rotation) can exert a disproportional effect on the "total angular momentum" equation -- just look at Pluto and the Sun.



Even now, Jupiter's four major satellites (which have collective masses approximately 1/10,000th of Jupiter itself), during the course of their complex orbital interactions, are historically known to cause *time-altered* behavior in a variety of well-known Jovian phenomena--

Including -- "anomalous" *latitude and longitude motions* of the Great Red Spot itself.

As we presented at the U.N. in 1992, the Great Red Spot -- a mysterious vortex located for over 300 years at that "infamous" 19.5 degrees S. Latitude, via the circumscribed tetrahedral geometry of the equally infamous "27 line problem" -- is *the* classic "hyperdimensional signature" of HD physics operating within Jupiter.

The existence of decades of recorded "anomalous motions" of this Spot, neatly *synchronized* with the highly predictable motions of Jupiter's own moons, are clearly NOT the result of conventional "gravitational" or "tidal" interactions -- in view of the relatively insignificant masses of the moons compared to Jupiter itself; but, following Maxwell and Whittaker, *the hyperdimensional effects of these same moons* -- via the long "lever" of angular momentum on the constantly changing, vorticular scalar stress potentials *inside* Jupiter -- that is a very different story ...

So, Hyperdimensional Test number three:

3) Look for small, short-term amplitude-variations in the infrared emission levels of all the giant planets ... synchronized (as are the still-mysterious motions of the GRS on Jupiter) with the orbital motions and conjunctions of their *moons*.

All NASA models for the "anomalous energy emissions" of these planets have assumed a steady output; the "snapshot" values derived from the mere *few hours* of Voyager fly-bys in the 1980's are now firmly listed in astronomy texts as new "planetary constants"; the reason: the emissions are viewed by NASA as either "primordial heat," stored across the aeons; energy release from internal long-term radioactive processes; or literal, slight *settling* of portions of the entire planet, still releasing gravitational potential energy ... all processes that will not change perceptibly even in *thousands* of years!

Confirmed *short-term variations* in the current planetary IR outputs, of "a few hours" (or even a few days) duration -- and *synchronized* with the orbital periods of the planets' satellites themselves -- would thus be stunning evidence that *all* the "mainstream" explanations are in trouble ... and that the Hyperdimensional Model deserves much closer scrutiny ...

In this same vein: unlike all "conventional NASA explanations," in a phenomenon akin to "hyperdimensional astrology," the HD model also specifically predicts significantly larger, long-term variability in these major planetary IR outputs ... of several *years* duration.

These (like the shorter variations triggered by the changing geometry between the satellites) should be caused by the *constantly changing hyperdimensional* (spatial stress) *interactions* between the major planets themselves ... as they continually change their geometry relative to one another, each orbiting the Sun with a different relative velocity.

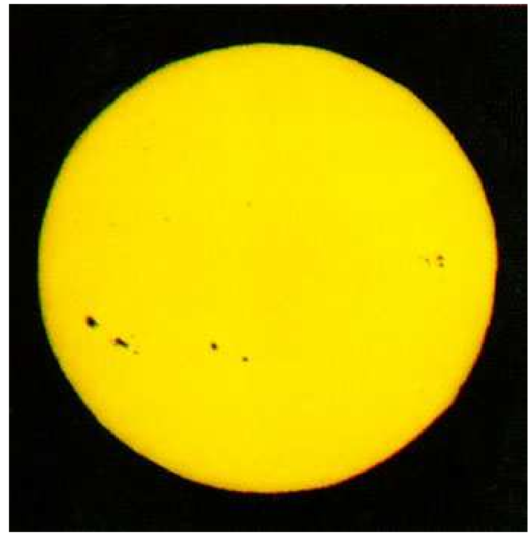
These changing interactive stresses in the "boundary between hyperspace and real' space" (in the Hyperdimensional Model) now also seem to be the answer to the mysterious "storms" that, from time to time, have suddenly appeared in the atmospheres of several of the outer planets. The virtual "disappearance," in the late 80's, of Jupiter's Great Red Spot is one remarkable example; Saturn's abrupt production of a major planetary "event," photographed by the Hubble Space Telescope in 1994 as a brilliant cloud erupting at 19.5 degrees N. (where else?!), is yet another.

Since the prevailing NASA view is that these planets' "excess" IR output *must* be constant over time, no one has bothered to look for any further correlations -- between a rising or falling internal energy emission ... and the (now, historically well-documented) semi-periodic eruptions of such "storms."

They should.

On Hyperdimensional Physics… and More….

Part III (back)



Which returns us to the Sun.

There is a very well known, long period, and still mysterious variability associated with the largest "hyperdimensional gate" in our own neighborhood -- our "local" star, the Sun.

Its complex changes, which include a host of related surface phenomena -- solar flares, coronal disturbances, mass ejections, etc. -- is termed "the sunspot cycle" ... because the number of simultaneous "spots" (lower-temperature vortices appearing "dark" against the hotter solar surface, as this activity occurs -- left) waxes and wanes over about 11 years. (The full magnetic reversal of the Sun's polarity takes two complete sunspot cycles to return to "zero" -- thus the complete "solar cycle" is about 20 years.)

In the 1940's, the Radio Corporation of America (RCA) hired a young electrical engineer -- John Nelson -- in an effort to improve the reliability of HF ("short-wave") radio communications around Earth. Such radio transmissions had been observed to be, for some reason, more reliable in the "lulls" in between, than during solar activity associated with "peak" sunspot years.

To his surprise, Nelson soon specifically correlated this rising and falling radio interference with not only sunspot cycle, but with the motions of the major planets of the solar system; he found, to his increasing astonishment, a very repeatable -- in essence, *astrological* correlation ... between the inexorable orbits of all the planets (but especially, Jupiter, Saturn, Uranus and Neptune -- which, remember, hold essentially all the solar' system's known *angular momentum*) ... and major radio-disturbing eruptions on the Sun!

Planetary Position Effect on Short-Wave Signal Quality

J. H. NELSON

A new approach to an as yet unsolved problem is the observance of planetary effects on trans-atlantic short-wave radio signals. Correlation over seven years shows that certain planetary arrangements agree well with the behavior of short-wave signals.

AT THE Central Radio Office of RCA Communications, Inc., in lower Manhattan, an observatory housing a 6-inch refracting telescope is maintained for the observation of sunspots. The 'purpose of erecting this observatory in 1946 was to develop a method of forecasting radio storms from the study of sunspots. After about one year of experimenting, a forecasting system of short-wave conditions was inaugurated based upon the age, position, classification, and activity of sunspots. Satisfactory results were obtained, but failure of this system from time to time, indicated that phenomena other than sunspots needed to be studied. The first article[1] by the

planets held a "multiple of 90 degrees" arrangement among themselves, the correlation was more pronounced. These arrangements were called "multiple configurations" and exist when two planets are at 0 degree with each other and a third planet is 90 degrees or 180 degrees away from them. Also, a multiple exists when two planets are separated by 180 degrees with a third planet 90 degrees from each. These multiples are quite common. A more uncommon type of multiple is the case where all three planets are at 0 degree with each other. From the few cases recorded, this type of multiple shows the least correlation.

The Hyperdimensional Model finally provides a comprehensive theoretical explanation -- a "linking mechanism" -- for these, to a lot of astronomers, still embarrassing decades-old RCA observations. For, in essence what John Nelson had *rediscovered* was nothing short of a "Hyperdimensional Astrology" (see photo below) -- the ultimate, very ancient, now highly demonstrable *angular momentum* foundations behind the *real* influences of the Sun and planets on our lives ...

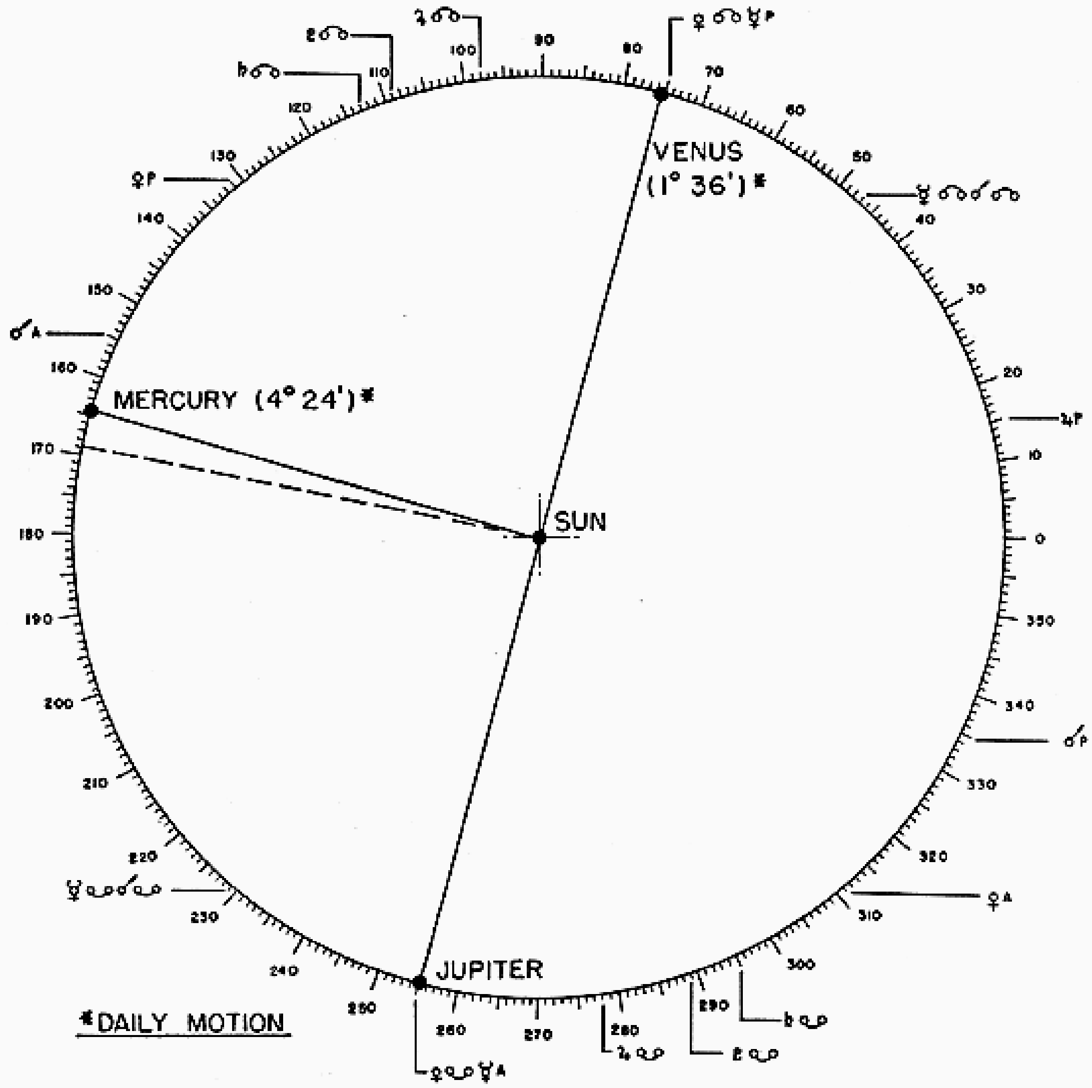


Figure 1. The heliocentric arrangement of Venus, Mercury, and Jupiter on February 23, 1948, which resulted in severe signal degradation on that day and the one following

and correlated with existing radio conditions. Multicycles for these three planets were found in nine cases which are listed in Table I. Case 10 is a triple multicycle that coincided with an extremely severe signal degradation in 1951. The heliocentric arrangements of the planets involved are shown in Figures 1 and 2 for Cases 1 and 10.

Single configurations (one cycle) between only two isolated planets show the least correlation, but often when several single cycles between several isolated planets coincide in time the correlation is quite pronounced. Most of the single cycles, however, do correlate with at least slight signal degradation. Some correlate with severe degradation.

At times, two, and sometimes three, complete multicycles occur in the space of a few days. At other times a multicycle will occur mingled with one or more single cycles between other planets.

Theoretically, if these planetary arrangements do have the effect that correlation indicates, the cycles between the slow planets should have gradual long-term effects establishing an over-all standard. The most degraded periods should come as the faster planets come into cycle with them or among themselves.

Jupiter and Saturn, the largest planets in the solar system, are the most important. Due to their great size and slow motion, they can exercise the predominating

For, as part of his solar research, Nelson also "rediscovered" something else ...

"It is worthy of note that in 1948 when Jupiter and Saturn were spaced by 120 degrees, and solar activity was at a maximum, radio signals averaged of far higher quality for the year than in 1951 with Jupiter and Saturn at 180 degrees and a considerable decline in solar activity. In other words, the average quality curve of radio signals *followed the cycle curve between Jupiter and Saturn rather than the sunspot curve* ... [emphasis added]."

-- J.H. Nelson, "Planetary Position Effect on Short-Wave Signal Quality"
(Electrical Engineering, May 1952)

These decades-old observations are very telling ... not only *confirming* Jupiter and Saturn as the primary "drivers" behind the Sun's known cycle of activity (in the HD Model), but strongly implying an additional *direct* effect of their changing angular relationship on *the electrical properties of Earth's ionosphere*. This, of course, is totally consistent with these changing planetary geometries affecting not just the Sun, but the other planets as well ... just as "conventional" astrologers have claimed -- via Maxwell's "changing scalar potentials"...

Therefore, at this point, *only* the hyperdimensional theory--

1) Points to the (literally!) the deepest implications of the simple astronomical fact that the "tail wags the dog"-- that the planets in this physics are fully capable of exerting a determinant influence on the Sun -- and each other -- through their disproportionate ratio of total solar system *angular momentum* ... over 100 to 1, in the [known!] planets' favor.

Only the Hyperdimensional Model--

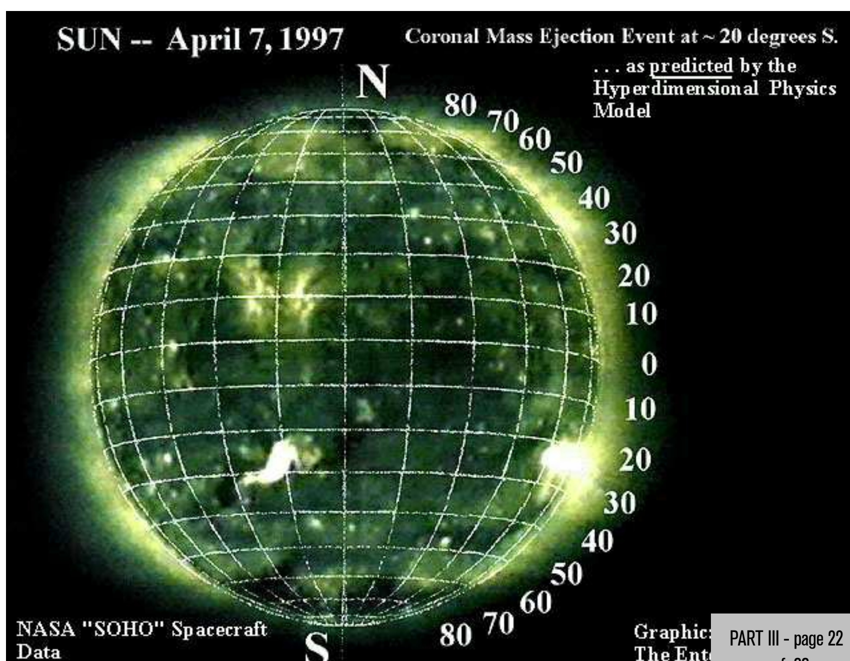
2) Possesses the precise *physical mechanism* -- via Maxwell's "changing quaternion scalar potentials" -- accounting for this "anomalous" planetary angular momentum influence.

And, only the HD theory--

3) Has already publicly identified, at the United Nations, in 1992, a blatant *geometric* clue to this entire HD solar process: the maximum sunspot numbers (those large, relatively "cool," *rotating vortices* appearing on the solar surface), rising and falling and methodically changing latitude, during the course of the familiar 22-year solar cycle--

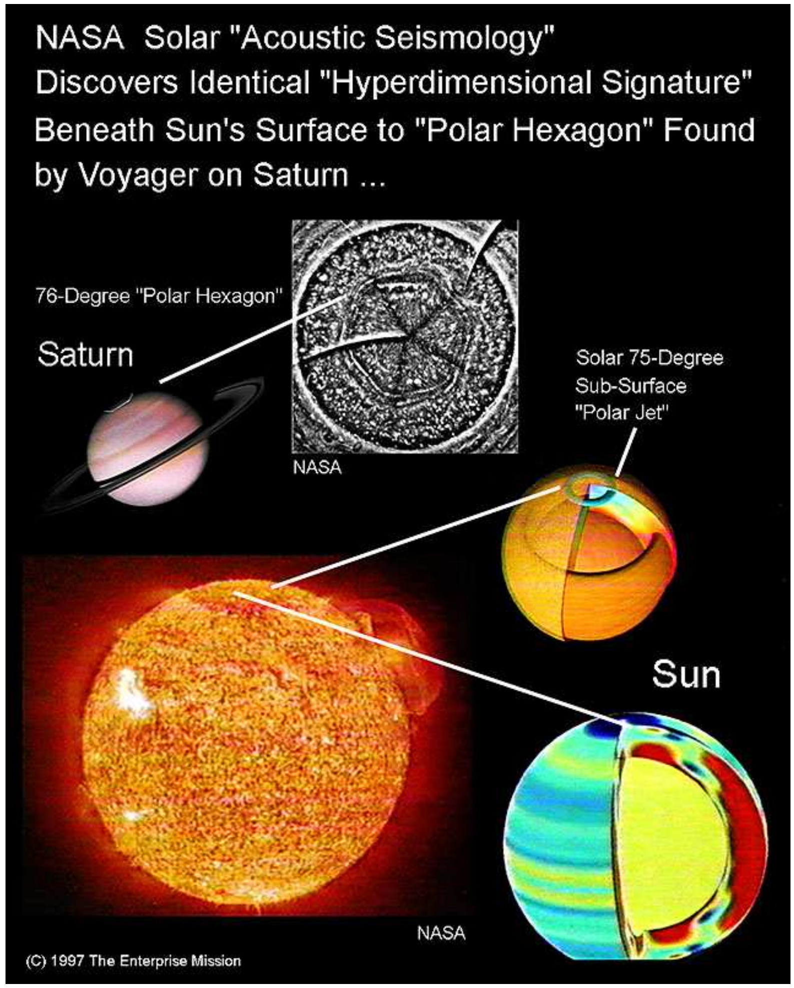
And peaking every half-cycle (~11 years), at the hyperdimensionally significant solar latitude of ~ "19.5 degrees!" (see photo right)

Furthermore, recent discoveries via the indirect technique of "helioseismology" (optical monitoring of sound waves vibrating back and forth within the Sun), have revealed another clear solar "hyper-dimensional signature at play; a curious "jet," flowing around the north pole several thousand miles below the visible solar surface; the remarkable similarity to an equivalent phenomenon



discovered by *Voyager* flowing around the north pole of Saturn -- a "polar hexagon" (see photo below) in the clouds -- even to the *latitude*, seems just a bit "too coincidental."

Unfortunately, because the discovery is not based on direct imaging (as with *Voyager*), but on an indirect "sub-surface flow" technique, the investigators have rounded off the corners of the potential sub-surface "solar hexagon" (see photo right); in fact, they should consult more frequently with their NASA planetary colleagues for additional examples of this geometric, now clearly *hyperdimensional* "flow pattern" elsewhere in the solar system ...



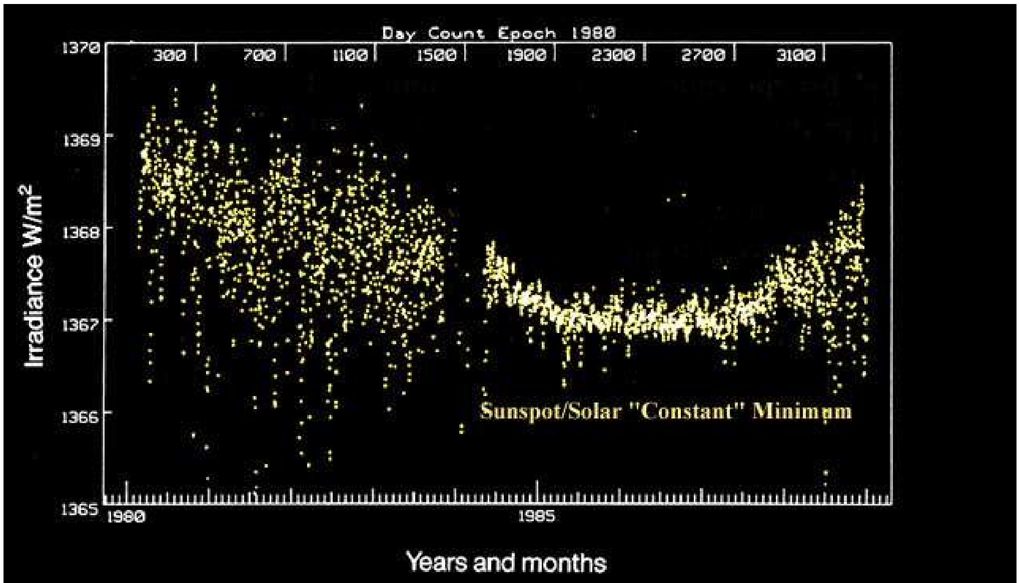
The increasing identification of the *hyperdimensional* mechanism underlying the Sun's primary energy production has, unfortunately, brought with it certain inevitable, potentially disquieting predictions ...

Described first by Kepler as his "Third Law," the farther out a planet orbits from its star the longer is its period of revolution. Since we're talking about possible *additional* planets driving this entire Hyperdimensional Solar Process (see again angular momentum diagram, above) -- planets that must be *hundreds of times* farther from the Sun than Earth (Pluto is "only" 40 times its distance, and orbits in "only" about 250 years) -- the "years" of these extremely distant worlds could equal thousands if not *tens of thousands* of Earth's year ... depending on their orbits. Because of these immense orbital periods, the cycles of solar energy production driven by their combined angular momentum will also be v-e-r-y long, indeed--

Perhaps as long as ~26,000 years ... (remember that number).

Also remember that such distant planets -- even if not particularly massive -- will have a disproportionately large effect on the total solar HD energy generation, because of the enormous "leverage" in the angular momentum equation with increasing distance. Thus, these still undiscovered worlds must in fact account for most of the solar energy we see ... depending on the actual orbital periods; the repeated ultra long-term phasings of their orbits -- creating equally long-term angular momentum resonances in the Sun -- must produce resulting long-term cyclic changes in the Sun's total luminosity ... lasting literally *thousands of years* ... far longer than the short-term, historical "sunspot cycle" Nelson first linked directly to the known members of the solar system.

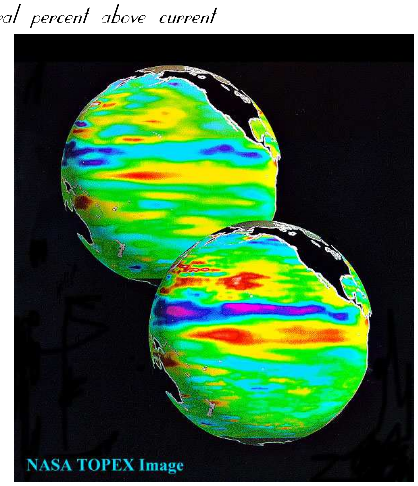
There is already a well-known link, between the historical 11-year "sunspot cycle" -- increased solar flares, x-ray emission, frequency of "coronal mass ejections," etc. -- and a measurable (if inexplicable as yet) increase in total *solar energy production*. Misnamed "the solar constant," (see below) this cyclic increase and decrease (according to satellite measurements) is curiously in phase with the current sunspot cycle ... averaging about 0.15%.



The solar irradiance measured by ACRIM over the period 1980–88, showing sunspot-associated dips at times of large spot activity (1980–84 and 1988 onwards) and the prolonged minimum (1985–87) when solar activity was low. The observational errors are larger for the period between the end of 1980 and the beginning of 1984 because of a lower rate of data acquisition and the coarse pointing of the *SMM* spacecraft in this interval. (Courtesy R. C. Willson and H. S. Hudson)

This NASA-documented short-term variation *of the entire Sun*, is now directly traceable (in the HD Model) to *the changing geometric phase relationships* between the solar system's two largest known planets, Jupiter and Saturn (as Nelson long ago confirmed); their orbital conjunctions -- when both worlds return again to the *same geometric position*, relative to one another -- take place on average roughly every 20 years ... the mean of the full "magnetic" solar cycle! (At the "half cycle" -- the familiar 11-year sunspot period itself -- Jupiter and Saturn are, of course, 180 degrees *out of phase* ... a critical clue to the determinate, modulating hyperspatial *geometry* actually inherent in this process ...)

But, if the known changes in solar output are due to hyperdimensional effects of the largest known planets, what of the magnitude of "aether stress" produced by our proposed "new planets" -- with angular momentum contributions *hundreds of times* greater?; the long-term cyclic increase in solar energy created by those cyclic phasings (yet-to-be-experienced in recorded history ...) could measure as much as *several percent above current solar output.* This is more than enough additional energy -- even without Mankind's current addition of significant "greenhouse gasses" to the atmosphere -- to trigger profound, millennia-long climatic changes here on Earth--



Including ... melting ice caps; rising ocean levels; dramatic changes in jet stream altitudes and activity; increased tornado intensities; increased hurricane wind velocities ... and--

A *permanent* "El Nino" (whose warmest waters, satellites report, are at ... *19.5 degrees*).

And that's saying nothing about HD energy added to the *internal* workings of our planet ...

From all indications, we are now well into *just such a new, long-term, cyclic solar period* ... just as the HD Model has predicted. The implications should be obvious.

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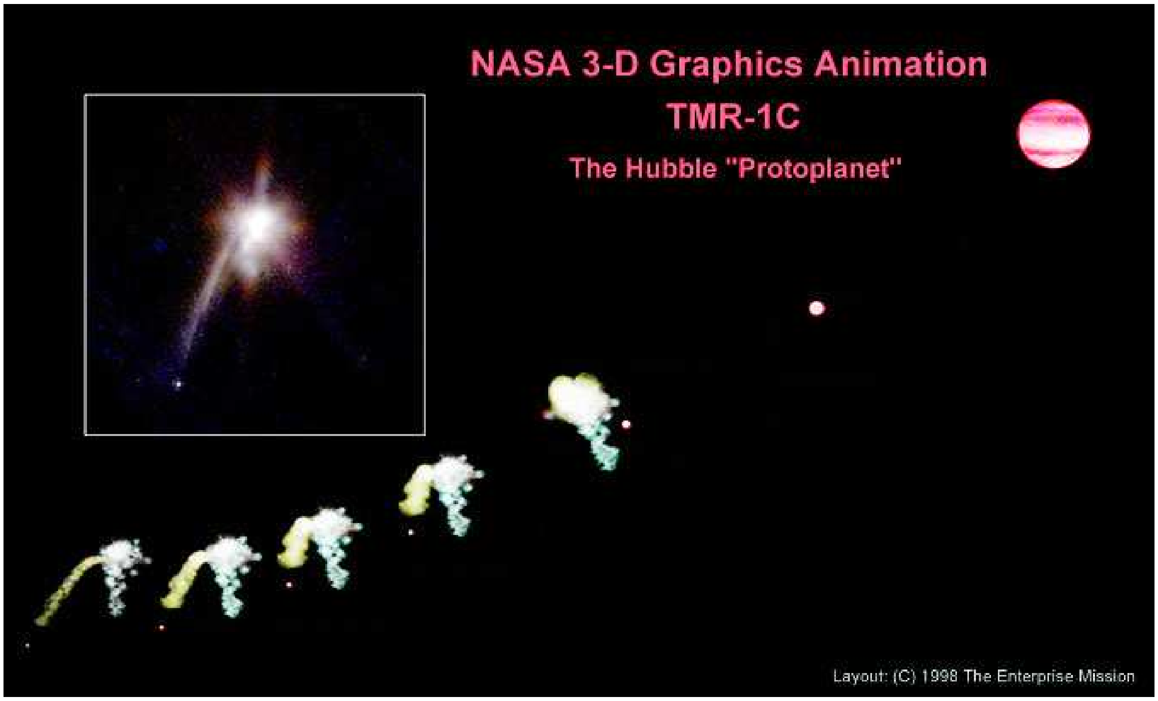
Enter once more, Susan Terebey.

With the discovery and official NASA announcement of Dr. Terebey's "new planet" -- a "Jovian-type" world 450 light years away ... yet, in apparent *total isolation* -- we have the perfect conditions for a …….

New series of crucial tests of the "Hyperdimensional Model," starting with--



1) Systematic spectroscopic observations (carried out over days or months), to detect the presence of any *satellites* of this proposed Jovian-type world. Curiously, this possibility has not even been alluded to in any of the official NASA presentations on this object (see NASA artist's depiction above).

It is almost inconceivable that such a giant world could have formed without multiple orbiting satellites; just look at the "miniature solar systems" of moons orbiting the major planets of *this* solar system ...

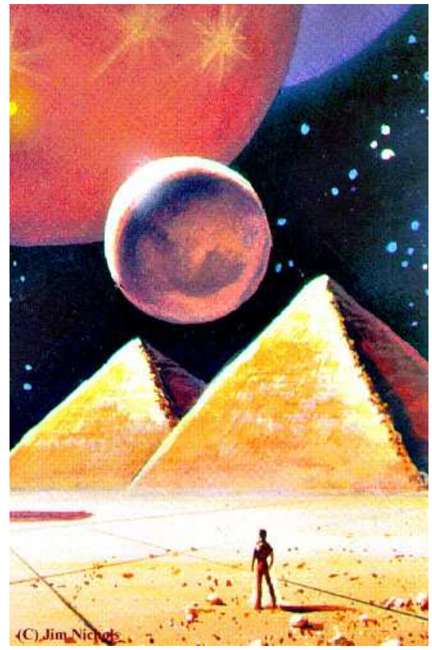
Confirmation of such additional objects would in itself be another major scientific "first." But, in this case, it would also have far deeper implications; some of those same moons could have considerable mass themselves -- perhaps as much as Mars ... or even *Earth* ... depending on the parent planet's mass. The latter could, in fact, be independently determined (via appropriate application of "Kepler's Laws") through physical detection of such moons and determining their *orbital periods*. This would then provide a completely independent calibration of the accuracy of the "primary object's" currently estimated mass -- derived, as Terebey's freely admits, solely from theoretical "cooling curves" (see above) and the "new planet's" measured IR luminosity itself.

These additional remarkable objects (if they exist ...) should be detectable with a variety of *present* astronomical technologies -- including Hubble -- provided someone *looks*.

NIBIRU

[And, confirmation of truly massive satellites (a "Mars" or "Earth") circling this new "potential planet" could vindicate, in a stroke, a major piece of Van Flandern's own revolutionary model -- that our own Mars (if not a number of other objects) once orbited a *larger planet* in this solar system as satellites ... until all (including Mars) were released by the catastrophic *explosion* of that "parent world."

Van Flandern, one of the world's leading celestial mechanics experts, has calculated the probable orbital parameters of such an extraordinary event -- and has reached the conclusion that Mars' uniquely elliptical path around the Sun (of all the inner planets) is highly consistent with its "escape" from such a "missing," former member of the solar system ...]



Successful detection of a "Mars" or "Earth" (or any significant satellites) orbiting Susan Terebey's "new planet" would immediate present possibilities for carrying out the same variety of HD tests proposed (above) for the outer planets of this solar system, starting with--

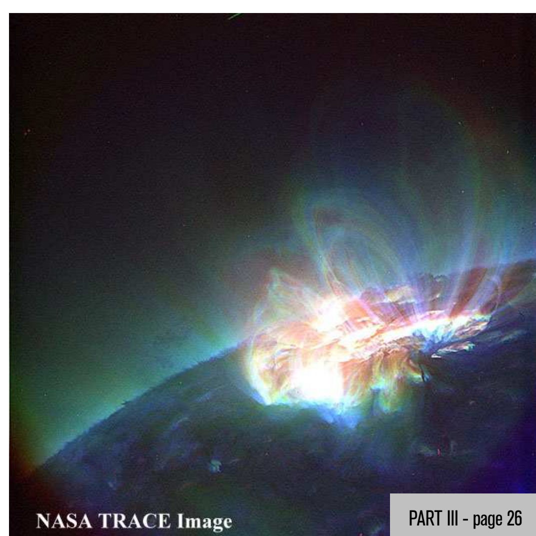
2) Detection of distinct *variations* in the "new planet's" own infrared emissions -- *synchronized* with the calculated orbital periods of any detected satellites (or resonances thereof ...). This would offer immediate, compelling evidence for the general correctness of the "Hyperdimensional Model" ... *especially*, if such IR signatures could be matched with similar types of time-varying emissions observed radiating from the giant planets of *this* solar system ...

Such confirmations would provide crucial and timely evidence supporting the basic correctness of the "HD Model." Equally important, if such confirmations are forthcoming, the extraordinary possibility will be further enhanced that this same fundamental physics, in our own solar system, could ... and *has--*

Destroyed *entire worlds ...*

For those who need a "practical" reason to address these issues, consider this:

Understanding the conditions under which these types of "epochal events" could come to pass ... and *if*, through proper understanding and control of "HD Physics," they can be averted for *our* planet ... would seem a simple, basic rationale for renewed interest in what Maxwell *really* stated. Given the demonstrable, historically-unprecedented changes currently occurring in our own environment -- from mysteriously-rising geophysical and volcanic activity (some of the most significant now occurring at that suspicious "19.5 degrees!"), to increasingly anomalous climatological and meteorological activity (does anyone notice that hurricanes have always been born at an average latitude of ...19.5 degrees?) -- verifying the effects of a *changing* "hyperdimensional physics" in our own neighborhood is far from being "merely academic."



Then, of course, there are the continuing, dramatic changes in the Sun...

Immediate solar predictions of the HD Model are simple and quite clear: increasingly violent average solar surface activity -- modulated with the rising and falling with the familiar "sunspot cycle" of ~11 years (cycle #23 should be a "lulu!") -- taking place against a backdrop of equally dramatic, long-term rising *of total solar energy emission.*

The major unknown is the overall effect of this slowly increasing "HD energy" availability throughout *the entire solar* *system*; for, remember, the changing "aether stress" is *not* localized inside stars or planets; these concentrations of "matter" merely allow us to trace the visible effects -- like red dye in clear water -- of these underlying, changing "hyperspatial strains." Thus, the overall *dynamical* effect of the Earth's own significant (to us!) angular momentum interaction with this changing solar system physics -- including its own long-term, "higher order rotation," the ~26,000 year *precessional cycle* and the mysterious role of the *Moon* -- is all still quite uncertain ...

However, recent anomalous observations of perhaps the most fundamental dynamical parameter of Earth -- *its own rotation* -- seem to support this growing perception that "something" indeed is rapidly changing in the solar system ...

For some inexplicable reason, there has been an accelerating slow-down of the Earth's spin on its own axis over the last twenty years -- as measured against electronic transitions of the "Cesium Atomic Clock," maintained at the National Bureau of Standards, in Boulder, Colorado.

A second makes a world of difference

By Bill Scanlon
Rocky Mountain News Staff Writer

BOULDER — At 5 p.m. New Year's Eve, a second will be added to the atomic clock here to slow it into synch with Earth's rotation.

Budget agreement or not, a physicist and a computer programmer from the National Institute for Standards and Technology in Boulder will be on hand Sunday when the world's clocks add a second.

Five o'clock here is midnight on the last day of the year, Greenwich, England, time. The official last minute of the year will be 61 seconds long.

Earth's rotation is slowing ever so slightly every year. In addition, the measure of a second — 9.193 billion oscillations of the cesium atom — wasn't quite precise when that became the official standard in 1967, said Judah Levine, physicist with NIST in Boulder.

So, every year or year-and-a-half, on Dec. 31 or June 30, the world's most accurate clocks add a second — called a leap second.

"The Earth and the clock get out of synch. We live by the Earth, and the Earth doesn't care one whit what the atom is doing," said Don Sullivan, chief of the time and frequency division at NIST in Boulder.

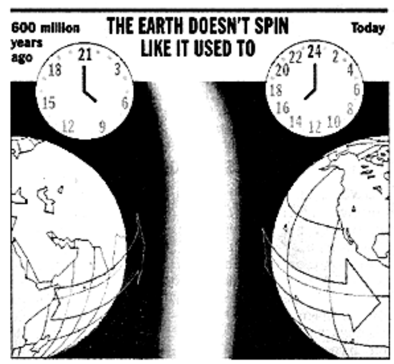
Although the leap second is needed mostly to adjust to imperfections in the atomic clock, and only marginally to adjust to a decelerating Earth, those seconds add up over time. Scientists at Australia's Adelaide University believe the Earth day was just 23 hours long at the time of the dinosaurs.

Levine and computer programmer Trudi Peppler are two of the handful of NIST employees working through the federal-government furlough. Chances are slim that anything would go wrong with the atomic clock if no one monitored it the next few days. There are batteries, generators and several back-up clocks to keep things humming.

But, lo, what a catastrophe if the fail-safe system failed:

- Navigators and geologists rely on the atomic clock for precise positioning. By measuring the time it takes a signal traveling at the speed of light to reach a satellite, an object's exact location can be determined.
- Without the atomic clock, long-distance phone calls and other telecommunications that use bands of optic fiber would be slowed or garbled.

"If clocks are synchronized, we can get a lot more information through the noise," Sullivan said.



Days have been getting longer. Geologists speculate that more than 600 million years ago a day was just 21 hours. Partly because the earth is rotating more slowly, the world's time experts will add a 'leap second' to the end of 1995.

Source: Adelaide University / National Institute of Standards & Technology. Graphic, Rocky Mountain News

Theoretically stable to "plus or minus one second *in a million years* ..." the official atomic clocks have literally *had to be adjusted by over 20 seconds* in these preceding two decades -- a stunning change by astronomical standards, and striking confirmation of some kind of major, "progressive phase-shift" now occurring, between the rotation of our planet and the atomic-level "constants" that govern the quantum standards of the Clock. This need to, with increasing frequency (now, approximately *every six months*) update a *full second* differential between "dynamical time" and "time at the atomic level," is profound confirmation of some kind of fundamental coupling between the *angular momentum* of our own planet, and the larger changes occurring in the hyperspatial physics of the solar system ... as outlined in the basic HD Model. This includes the possibility that these changing "hyperspatial stresses," due to the progressive orbital movement of our as-yet-undiscovered "outer planets" (in the Model) are simply causing increasing "Maxwellian scalar potential changes" all across the solar system *at the atomic level* -- changing (via the "Aharonov-Bohm Effect") *all* resultant quantum-level "constants" governing the clocks by small (but laboratory-measurable) amounts.

If this is true, it could thus be the Clocks themselves that are also *internally* changing ... *simultaneous* with the predicted, accelerating slow-down in the basic rotation of the Earth!

Additional evidence that "something major" is occurring in the solar system, is the recent announcement (by three major "world-class" laboratories) of startling changes in *another* fundamental physical parameter -- the Gravitational Constant; according to an article published in "Science News" in 1995, not only has this centuries-old "constant" been found to differ now from all previous "textbook" values ... each of the three laboratories reported *different* changes ... the largest amounting to a whopping (by measurement standards) 0.06%!

SCIENCE NEWS -- April 29, 1995

Gravity's force: Chasing an elusive constant

Determining the values of fundamental physical constants has long served as a test of both physical theory and measurement technology. Now, experiments by three independent groups have produced values for the strength of the gravitational force (G) that disagree significantly with the currently accepted number and with each other (see table).

The teams involved in these experiments reported their results at last week's American Physical Society meeting, held in Washington, D.C.

"Each of these groups has done a careful job, but G is an extremely hard number to nail down," says George T. Gillies of the University of Virginia in Charlottesville.

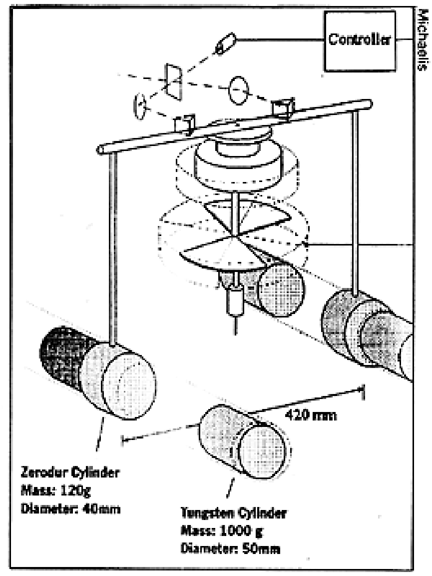
The problem stems from the fact that gravity is much weaker than the other

Luther

tric force compensates for the gravitational force between pairs of masses (see diagram). In this case, the researchers eliminated the complicating effects of the wire by hanging metal cylinders from either end of a beam and letting the beam's support float in a mercury bath.

Hinrich Meyer and his colleagues at the University of Wuppertal in Germany adopted a different approach. This group used microwave technology to measure minute changes in the oscillation frequencies of a pair of pendulums disturbed by the movement of large masses in their vicinity.

The fact that these three, carefully performed experiments give different results "is truly a scientific mystery," comments Eric G. Adelberger of the University of Washington in Seattle. However, it doesn't necessarily imply that the physical theory is faulty.



Torsion balance used by Michaelis and his colleagues.

Completely inexplicable by any mainstream theories, such changes are intrinsically expected (ala Whittaker's derivation of "electrogravitic" linkages between gravity and light) in the true Maxwellian analysis of "varying scalar potentials" of the vacuum. Again: changes modulated by the *changing phase relationships* between our proposed "undiscovered outer planets" in the "total solar system angular momentum equation."

There is insufficient room to develop here (let alone to document) *all* the additional implications stemming from this Model (that will be done elsewhere). Suffice it to say, this is intrinsically a *changing physics*, affecting every known system of astronomical, physical, chemical and biological interaction differently over time -- because it affects the underlying, dynamical *hyperspace* foundation of "physical reality" itself ... starting with this solar system; that is *implicit* in the Model.

And now, according to all accumulating evidence and this centuries-old physics ... we are simply entering once again (after "only" 13,000 years ...) a phase of this recurring, grand solar system cycle *"of renewed hyperdimensional restructuring of that reality ..."*

It is for these basic reasons that NASA must now openly -- and rapidly -- carry out these recommended new observations of the solar system, including additional, detailed measurements of "TMR-1C" ... and then *immediately* tell us the results.

Time is getting short.

Appendix (back)

E.T. Whittaker

On the Partial Differential Equations of Mathematical Physics

WHITTAKER PAPER 1903

E.T. Whittaker

On the Partial Differential Equations of Mathematical Physics

Mathematische Annalen

Vol. 57, 1903, p. 333-355.

On the partial differential equations of mathematical physics.

By

E. T. Whittaker in Cambridge.

§ 1.

Introduction.

The object of this paper is the solution of Laplace's potential equation

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = 0,$$

and of the general differential equation of wave-motions

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = k^2 \frac{\partial^2 V}{\partial t^2},$$

and of other equations derived from these.

In § 2, the general solution of the potential equation is found.

In § 3, a number of results are deduced from this, chiefly relating to particular solutions of the equation, and expansions of the general solution in terms of them.

In § 4, the general solution of the differential equation of wave-motions is given.

In § 5, a number of deductions from this general solution is given, including a theorem to the effect that any solution of this equation can be compounded from simple uniform plane waves, and an undulatory explanation of the propagation of gravitation.

§ 2.

The general solution of the potential equation.

We shall first consider the equation

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = 0,$$

which was originally given by Laplace*).

*) *Mémoire sur la theorie de l'anneau de Saturne*, 1787.

Purposely Omitted

The question now naturally suggests itself, whether the most general solution of Laplace's equation can be represented by an expression of this type. We shall shew that the answer to this is in the affirmative.

For let $V(x, y, z)$ be any solution (single-valued or many-valued) of the equation

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = 0.$$

Let (x_0, y_0, z_0) be some point at which some branch of the function $V(x, y, z)$ is regular. Then if we write

$$x = x_0 + X, \quad y = y_0 + Y, \quad z = z_0 + Z$$

it follows that for all points situated within a finite domain surrounding the point (x_0, y_0, z_0), this branch of the function $V(x, y, z)$ can be expanded in an absolutely and uniformly convergent series of the form

$$V = a_0 + a_1 X + b_1 Y + c_1 Z + a_2 X^2 + b_2 Y^2 + c_2 Z^2 + d_2 YZ$$
$$+ e_2 ZX + f_2 XY + a_3 X^3 + \cdots.$$

Substituting this expansion in Laplace's equation, which can be written

$$\frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2} + \frac{\partial^2 V}{\partial Z^2} = 0,$$

and equating to zero the coefficients of the various powers of X, Y and Z, we obtain an infinite number of linear relations, namely

$$a_2 + b_2 + c_2 = 0, \text{ etc.}$$

between the constants in the expansion.

There are $\frac{1}{2}n(n-1)$ of these relations between the $\frac{1}{2}(n+1)(n+2)$ coefficients of the terms of any degree n in the expansion of V; so that only $\left\{\frac{1}{2}(n+1)(n+2) - \frac{1}{2}n(n-1)\right\}$ or $(2n+1)$ of the coefficients of terms of degree n in the expansion of V are really independent. It follows that the terms of degree n in V must be a linear combination of $(2n+1)$ linearly independent particular solutions of Laplace's equation, which are of degree n in X, Y, Z.

To find these solutions, consider the expansion of the quantity

$$(Z + iX\cos u + iY\sin u)^n$$

as a sum of sines and cosines of multiples of u, in the form

$$(Z + iX\cos u + iY\sin u)^n = g_0(X, Y, Z) + g_1(X, Y, Z)\cos u$$
$$+ g_2(X, Y, Z)\cos 2u + \cdots + g_n(X, Y, Z)\cos nu$$
$$+ h_1(X, Y, Z)\sin u + h_2(X, Y, Z)\sin 2u + \cdots$$
$$+ h_n(X, Y, Z)\sin nu.$$

Now $g_m(X, Y, Z)$ and $h_m(X, Y, Z)$ are together characterised by the fact that the highest power of Z contained in them is Z^{n-m}; moreover $g_m(X, Y, Z)$ is an even function of Y, whereas $h_m(X, Y, Z)$ is an odd function of Y; and hence the $(2n+1)$ quantities

$$g_0(X, Y, Z),\ g_1(X, Y, Z), \cdots, h_n(X, Y, Z)$$

are linearly independent of each other; and they are clearly homogeneous polynomials of degree n in X, Y, Z; and each of them satisfies Laplace's equation, since the quantity $(Z+iX\cos u+iY\sin u)^n$ does so. They may therefore be taken as the $(2n+1)$ linearly independent solutions of degree n of Laplace's equation.

Now since by Fourier's Theorem we have the relations

$$g_m(X, Y, Z)=\frac{1}{\pi}\int_0^{2\pi}(Z+iX\cos u+iY\sin u)^n\cos mu\, du,$$

$$h_m(X, Y, Z)=\frac{1}{\pi}\int_0^{2\pi}(Z+iX\cos u+iY\sin u)^n\sin mu\, du,$$

it follows that each of these $(2n+1)$ solutions can be expressed in the form

$$\int_0^{2\pi}f(Z+iX\cos u+iY\sin u,\ u)\, du$$

and therefore any linear combination of these $(2n+1)$ solutions can be expressed in this form. That is, the terms of any degree n in the expansion of V can be expressed in this form; and therefore V itself can be expressed in the form

$$\int_0^{2\pi}F(Z+iX\cos u+iY\sin u, u)du,$$

or

$$\int_0^{2\pi}F(z+ix\cos u+iy\sin u-z_0-ix_0\cos u-iy_0\sin u,\ u)\, du,$$

or

$$\int_0^{2\pi}f(z+ix\cos u+iy\sin u,\ u)\, du,$$

since the $z_0+ix_0\cos u+iy\sin u$ can be absorbed into the second argument u.

Now V was taken to be any solution of Laplace's equation, with no restriction beyond the assumption that some branch of it was at some point a regular function — an assumption which is always tacitly made in the solution of differential equations; and thus we have the result, that *the general solution of Laplace's equation*

$$\frac{\partial^2 V}{\partial x^2}+\frac{\partial^2 V}{\partial y^2}+\frac{\partial^2 V}{\partial z^2}=0$$

is

$$V=\int_0^{2\pi} f(z+ix\cos u+iy\sin u,\, u)\,du,$$

where f is an arbitrary function of the two arguments

$$z+ix\cos u+iy\sin u \quad \textit{and} \quad u.$$

Moreover, it is clear from the proof that no generality is lost by supposing that f is a periodic function of u.

This Theorem is the three-dimensional analogue of the theorem that the general solution of the equation

$$\frac{\partial^2 V}{\partial x^2}+\frac{\partial^2 V}{\partial y^2}=0$$

is

$$V=f(x+iy)+g(x-iy).$$

§ 3.

Deductions from the Theorem of § 2; Particular Solutions; Expansions of the General Solution.

1°. *Interpretation of the solution.* We may give to the general solution just obtained a concrete interpretation, as follows.

Since a definite integral can be regarded as the limit of a sum, we can regard V as the sum of an infinite number of terms, each of the type

$$V_r=f_r(z+ix\cos u_r+iy\sin u_r)$$

each term corresponding to some value of u_r.

But this term is a solution of the equation

$$\frac{\partial^2 V_r}{\partial X_r^2}+\frac{\partial^2 V_r}{\partial Z_r^2}=0,$$

where

$$X_r= \quad x\cos u_r+y\sin u_r,$$
$$Y_r=-x\sin u_r+y\cos u_r,$$
$$Z_r=z,$$

so that (X_r, Y_r, Z_r) represent coordinates derived from (x, y, z) by a rotation of the axes through an angle u_r round the axis of z.

Thus we see that *the general solution of Laplace's equation can be regarded as the sum of an infinite number of elementary constituents* V_r, *each constituent being the solution of an equation*

$$\frac{\partial^2 V_r}{\partial X_r^2} + \frac{\partial^2 V_r}{\partial Z_r^2} = 0,$$

and the axes (X_r, Y_r, Z_r) being derived from the axes (x, y, z) by a simple rotation round the axis of z.

2°. *The particular solutions in terms of Legendre functions.* It is interesting to see how the well-known particular solutions of Laplace's equation in terms of Legendre functions can be obtained as a case of the solution given in § 2.

The particular solutions in question are of the form

$$r^n P_n^m(\cos\theta)\cos m\varphi \quad \text{and} \quad r^n P_n^m(\cos\theta)\sin m\varphi$$
$$(n = 0, 1, 2, \cdots, \infty;\ m = 0, 1, 2, \cdots, n),$$

where (r, θ, φ) are the polar coordinates corresponding to the rectangular coordinates (x, y, z), and where

$$P_n^m(\cos\theta) = \frac{(-1)^m \sin^m\theta}{2^n n!}\,\frac{d^{n+m}(\sin^{2n}\theta)}{d(\cos\theta)^{n+m}}.$$

Now the function $P_n^m(\cos\theta)$ can be expressed by the integral

$$P_n^m(\cos\theta) = \frac{(n+m)(n+m-1)\cdots(n+1)}{\pi}(-1)^{\frac{m}{2}}\int_0^{2\pi}(\cos\theta + i\sin\theta\cos\psi)^n\cos m\psi\,d\psi$$

and thus we have

$$r^n P_n^m(\cos\theta)\cos m\varphi = \frac{(n+m)(n+m-1)\cdots(n+1)}{\pi}(-1)^{\frac{m}{2}}\int_0^{\pi}\left(z + i\sqrt{(x^2+y^2)}\cos\psi\right)^n \cos m\psi\cos m\varphi\,d\psi$$

$$= \frac{(n+m)(n+m-1)\cdots(n+1)}{2\pi}(-1)^{\frac{m}{2}}\int_0^{2\pi}\left(z + i\sqrt{x^2+y^2}\cos\psi\right)^n\cos m(\psi-\varphi)\,d\psi$$

$$= \frac{(n+m)(n+m-1)\cdots(n+1)}{2\pi}(-1)^{\frac{m}{2}}\int_0^{2\pi}(z + ix\cos u + iy\sin u)^n\cos mu\,du.$$

We see therefore that *the solution* $r^n P_n^m(\cos\theta)\cos m\varphi$ *is a numerical multiple of*

$$\int_0^{2\pi}(z + ix\cos u + iy\sin u)^n\cos mu\,du.$$

Similarly the solution $r^n P_n^m(\cos\theta)\sin m\varphi$ is a numerical multiple of

$$\int_0^{2\pi}(z+ix\cos u+iy\sin u)^n\sin mu\,du.$$

From this it is clear that *in order to express any solution*

$$\int_0^{2\pi}f(z+ix\cos u+iy\sin u,\,u)\,du$$

of Laplace's equation, as a series of harmonic terms of the form

$$r^n P_n^m(\cos\theta)\cos m\varphi \quad \textit{and} \quad r^n P_n^m(\cos\theta)\sin m\varphi,$$

it is only necessary to expand the function f as a Taylor series with respect to the first argument $z+ix\cos u+iy\sin u$, and as a Fourier series with respect to the second argument u.

As an example of this procedure, we shall suppose it required to find the potential of a prolate spheroid in the form

$$\int_0^{2\pi}f(z+ix\cos u+iy\sin u,\,u)\,du,$$

and to expand this potential as a series of harmonics. Let

$$\frac{x^2+y^2}{a^2}+\frac{z^2}{c^2}=0$$

be the equation of the surface of the spheroid; and suppose that it is a homogeneous attracting body of mass M. To find its potential, we can make use of the theorem that the potential at external points is the same as that of a rod joining the foci, of line-density $\frac{3M(c^2-a^2-z^2)}{4(c^2-a^2)^{\frac{3}{2}}}$; that is, it is

$$\frac{3M}{8\pi(c^2-a^2)^{\frac{3}{2}}}\int_0^{2\pi}du\int_{-\sqrt{c^2-a^2}}^{\sqrt{c^2-a^2}}\frac{(c^2-a^2-\zeta^2)\,d\zeta}{z-\zeta+ix\cos u+iy\sin u}$$

or

$$\frac{3M}{8\pi(c-a)^{\frac{3}{2}}}\int_0^{2\pi}\left\{(c^2-a^2-B^2)\log\frac{B+\sqrt{c^2-a^2}}{B-\sqrt{c^2-a^2}}+2\sqrt{c^2-a^2}\,B\right\}du,$$

where B is written for $z+ix\cos u+iy\sin u$.

Expanding the integrand in ascending powers of $\frac{1}{B}$, we have the potential in the form

$$\frac{3M}{2\pi}\int_0^{2\pi}\left\{\frac{1}{1\cdot3\cdot B}+\frac{c^2-a^2}{3\cdot5\cdot B^3}+\frac{(c^2-a^2)^2}{5\cdot7\cdot B^5}+\cdots\right\}du.$$

Since

$$\frac{1}{2\pi}\int_0^{2\pi}\frac{du}{B^{n+1}} = \frac{P_n(\cos\theta)}{r^{n+1}},$$

this gives *the required expansion of the potential* of the spheroid in Legendre functions, namely the series

$$3M\left\{\frac{1}{1\cdot 3r} + \frac{(c^2-a^2)P_2(\cos\theta)}{3\cdot 5\cdot r^3} + \frac{(c^2-a^2)^2 P_4(\cos\theta)}{5\cdot 7\cdot r^5} + \cdots\right\}.$$

This result may be extended to the case of the potential of an ellipsoid with three unequal axes, by using a formula for the potential of an ellipsoid given by Laguerre*)

3°. *The particular solutions of Laplace's equation which involve Bessel functions.* We shall next shew how the well-known particular solutions of Laplace's equation in terms of Bessel functions can be obtained as a case of the general solution. The particular solutions in question are of the form

$$e^{kz}J_m(k\varrho)\cos m\varphi \quad \text{and} \quad e^{kz}J_m(k\varrho)\sin m\varphi,$$

where k and m are constants, and z, ϱ, φ are the cylindrical co-ordinates corresponding to the rectangular co-ordinates x, y, z, so that

$$x = \varrho\cos\varphi, \qquad y = \varrho\sin\varphi.$$

Now if in the solution

$$e^{kz}J_m(k\varrho)\cos m\varphi$$

we replace $J_m(k\varrho)$ by its value

$$J_m(k\varrho) = \frac{1}{\pi}\int_0^{\pi}\cos(m\theta - k\varrho\sin\theta)\,d\theta,$$

we find after a few simple transformations that

$$e^{kz}J_m(k\varrho)\cos m\varphi = \frac{(-1)^{\frac{m}{2}}}{2\pi}\int_0^{2\pi}e^{k(z+ix\cos u+iy\sin u)}\cos mu\,du.$$

The other solutions which involve $\sin m\varphi$, can be similarly expressed: we see therefore that *the solutions*

$$e^{kz}J_m(k\varrho)\cos m\varphi \quad \textit{and} \quad e^{kz}J_m(k\varrho)\sin m\varphi$$

*) C. R., 1878.

are numerical multiples of

$$\int_0^{2\pi} e^{k(z+ix\cos u+iy\sin u)}\cos mu\,du$$

and

$$\int_0^{2\pi} e^{k(z+ix\cos u+iy\sin u)}\sin mu\,du$$

respectively. It follows from this that *in order to express any solution*

$$\int_0^{2\pi} f(z+ix\cos u+iy\sin u,\,u)\,du$$

of Laplace's equation as a sum of terms of the form

$$e^{kz}J_m(k\varrho)\cos m\varphi \quad \textit{and} \quad e^{kz}J_m(k\varrho)\sin m\varphi,$$

it is only necessary to expand the function f in terms of exponentials of its first argument $z+ix\cos u+iy\sin u$, and as a Fourier series with respect to its second argument u.

As an example of the use which may be made of these results, we shall suppose it required to express the potential-function

$$V=1+e^{-z}J_0(\varrho)+e^{-2z}J_0(2\varrho)+e^{-3z}J_0(3\varrho)+\cdots$$

(where z is supposed positive) as a series of harmonic terms of the type involving Legendre functions: and also to find a distribution of attracting matter of which this in the potential. This can be done in the following way. We have

$$\begin{aligned} V &= 1+e^{-z}J_0(\varrho)+e^{-2z}J_0(2\varrho)+e^{-3z}J_0(3\varrho)+\cdots \\ &= \frac{1}{2\pi}\int_0^{2\pi}\{1+e^{-z-ix\cos u-iy\sin u}+e^{-2(z+ix\cos u+iy\sin u)}+\cdots\}\,du \\ &= \frac{1}{2\pi}\int_0^{2\pi}\frac{du}{1-e^{-(z+ix\cos u+iy\sin u)}}. \end{aligned}$$

But if t be any variable different from zero, and such that $|t|<2\pi$, we have

$$\frac{1}{1-e^t}=-\frac{1}{t}+\frac{1}{2}-B_1\frac{t}{2!}+B_2\frac{t^3}{4!}-B_3\frac{t^5}{6!}+\cdots,$$

where B_1, B_2, are Bernoulli's numbers. Therefore, so long as z is positive and $|z+ix\cos u+iy\sin u|<2\pi$ i. e., so long as z is positive and $x^2+y^2+z^2<4\pi^2$ we have

$$V = \frac{1}{2\pi}\int_0^{2\pi}\left\{\frac{1}{z + ix\cos u + iy\sin u} + \frac{1}{2} + \frac{B_1}{2!}(z + ix\cos u + iy\sin u) + \cdots\right\} du$$

or

$$V = \frac{1}{r} + \frac{1}{2} - \frac{B_1}{2!} rP_1(\cos\vartheta) - \frac{B_2}{4!} r^3 P_3(\cos\theta) + \frac{B_3}{6!} r^5 P_5(\cos\theta) + \cdots$$

and *this is the required expansion of V as a series of harmonics involving Legendre functions.*

Next, since

$$\frac{1}{1 - e^{-z}} = \frac{1}{2} + \frac{1}{z} + \sum_{n=1}^{\infty} \frac{1}{z + 2ni\pi} + \frac{1}{z - 2ni\pi},$$

we have

$$V = \frac{1}{2\pi}\int_0^{2\pi} du \left[\frac{1}{2} + \frac{1}{z + ix\cos u + iy\sin u} + \sum_{n=1}^{\infty}\left\{\frac{1}{z + ix\cos u + iy\sin u + 2ni\pi} + \frac{1}{z + ix\cos u + iy\sin u - 2ni\pi}\right\}\right],$$

or

$$V = \frac{1}{2} + \frac{1}{\sqrt{x^2 + y^2 + z^2}} + \sum_{n=1}^{\infty}\left\{\frac{1}{\sqrt{x^2 + y^2 + (z + 2ni\pi)^2}} + \frac{1}{\sqrt{x^2 + y^2 + (z - 2ni\pi)^2}}\right\},$$

and therefore V *can be regarded as the potential due to a set of attracting masses placed at equal imaginary intervals $2i\pi$ along the axis of z.*

§ 4.

The differential equation $\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = k^2 \frac{\partial^2 V}{\partial t^2}$.

We shall next consider the general differential equation of wave-motions,

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = k^2 \frac{\partial^2 V}{\partial t^2},$$

where k is a constant.

Writing kt for t, this becomes

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = \frac{\partial^2 V}{\partial t^2},$$

which we shall take for the present as the standard form of the equation.

In order to find the general solution of this equation, we follow a procedure analogous to that of § 2. Let $V(x, y, z, t)$ be any solution (single-valued or many-valued) of the equation; and let (x_0, y_0, z_0, t_0) be a place at which some branch of the function V is regular. Then if we write $x = x_0 + X$, $y = y_0 + Y$, $z = z_0 + Z$, $t = t_0 + T$, it will be possible to expand this branch of the function V as a power-series of the form

$$V = a_0 + a_1 X + b_1 Y + c_1 Z + d_1 T + a_2 X^2 + b_2 Y^2 + c_2 Z^2 + d_2 T^2 + e_2 XY \\ + f_2 XZ + g_2 XT + h_2 YZ + k_2 YT + l_2 ZT + a_3 X^3 + \cdots,$$

which will be absolutely and uniformly convergent for a certain finite domain of values of X, Y, Z, T. Substituting this expansion in the differential equation, which may be written

$$\frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2} + \frac{\partial^2 V}{\partial Z^2} = \frac{\partial^2 V}{\partial T^2},$$

and equating to zero the coefficients of various powers of X, Y and Z, we obtain an infinite number of linear relations, namely

$$a_2 + b_2 + c_2 = d_2, \text{ etc.},$$

between the constants in the expansion. There are $\frac{1}{6}(n-1)\,n(n+1)$ of these relations between the $\frac{1}{6}(n+1)(n+2)(n+3)$ coefficients of terms of any degree n in the expansion of V; so that only

$$\frac{1}{6}\{(n+1)(n+2)(n+3) - (n-1)\,n(n+1)\}$$

or

$$(n+1)^2$$

of the coefficients of terms of degree n in the expansion of V are really independent. It follows that the terms of degree n in V must be a linear combination of $(n+1)^2$ linearly independent particular solutions of degree n in X, Y, Z, T.

To find these solutions, consider the expansion of the quantity

$$(X \sin u \cos v + Y \sin u \sin v + Z \cos u + T)^n.$$

If we first take the expansion in the form

$$g_0 + g_1 \cos v + g_2 \cos 2v + \cdots + g_n \cos nv \\ + h_1 \sin v + h_2 \sin 2v + \cdots + h_n \sin nv,$$

we have seen in § 2 that $g_0, g_1, \cdots, g_n, h_1, \cdots, h_n$, are linearly independent functions of X, Y, Z and T. Moreover, g_m and h_m are of the form $\sin^m u \times$ a polynomial of degree $(n-m)$ in $\cos u$, and therefore each of them contains $(n-m+1)$ independent polynomials in X, Y, Z, T. Thus the total number of independent polynomials in X, Y, Z, T, in the expansion of

$$(X \sin u \cos v + Y \sin u \sin v + Z \cos u + T)^n$$

in sines and cosines of multiples of u and v, is

$$(n+1)+2n+2(n-1)+2(n-2)+\cdots+2$$

or

$$(n+1)^2.$$

Now each of these polynomials must satisfy the equation

$$\frac{\partial^2 V}{\partial x^2}+\frac{\partial^2 V}{\partial y^2}+\frac{\partial^2 V}{\partial z^2}=\frac{\partial^2 V}{\partial t^2},$$

since the quantity

$$(X\sin u\cos v+Y\sin u\sin v+Z\cos u+T)^n$$

does so: and therefore they may be taken as the $(n+1)^2$ linearly independent solutions of the equation

$$\frac{\partial^2 V}{\partial x^2}+\frac{\partial^2 V}{\partial y^2}+\frac{\partial^2 V}{\partial z^2}=\frac{\partial^2 V}{\partial t^2}$$

which are homogeneous of degree n in X, Y, Z, T. Now by Fourier's theorem we have

$$g_m=\frac{1}{\pi}\int_0^{2\pi}(X\sin u\cos v+Y\sin u\sin v+Z\cos u+T)^n\cos mv\,dv;$$

and since g_m is of the form

$$\sum_{r=0}^{n-m}u_r\sin^m u\cos^r u,$$

where u_r is one of the polynomials in question, it is clear that g_m can be expressed as a sum of sines or cosines of multiples of u, according as m is even or odd; and the coefficient of one of these sines or cosines, say of $\cos su$, is

$$\frac{2}{\pi}\int_0^{\pi}g_m\cos su\,du.$$

It follows that each of the polynomials u_r can be expressed in the form

$$\int_0^{\pi}g_m f(u)\,du,$$

where $f(u)$ denotes some periodic function of u; that is, it can be expressed in the form

$$\int_0^{2\pi}\int_0^{\pi}(X\sin u\cos v+Y\sin u\sin v+Z\cos u+T)^n f(u)\cos mv\,du\,dv.$$

It follows from this that each of the $(n+1)^2$ polynomial solutions of degree n can be expressed in the form

$$\int_0^{2\pi}\int_0^{\pi}(X \sin u \cos v + Y \sin u \sin v + Z \cos u + T)^n f(u, v)\, du\, dv,$$

where $f(u, v)$ denotes some periodic function of u and v; and therefore the terms of degree n in V can be expressed in this form.

The function V itself can therefore be expressed in the form

$$\int_0^{2\pi}\int_0^{\pi} f(X \sin u \cos v + Y \sin u \sin v + Z \cos u + T, u, v)\, du\, dv,$$

where f denotes some function of the three arguments

$$X \sin u \cos v + Y \sin u \sin v + Z \cos u + T,\ u,\ \text{and}\ v;$$

and f may without loss of generality be supposed to be periodic in u and v.

Now

$$\begin{aligned} &X \sin u \cos v + Y \sin u \sin v + Z \cos u + T \\ &= (x\ \sin u \cos v + y\ \sin u \sin v + z\ \cos u + t) \\ &- (x_0 \sin u \cos v + y_0 \sin u \sin v + z_0 \cos u + t_0); \end{aligned}$$

and the termo

$$(x_0 \sin u \cos v + y_0 \sin u \sin v + z_0 \cos u + t_0)$$

can be absorbed into the arguments u and v; moreover V was taken to be any solution of the partial differential equation; we have, therefore, on writing $\frac{t}{k}$ for t, the result that *the general solution of the partial differential equation of wave-motions,*

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = k^2 \frac{\partial^2 V}{\partial t^2},$$

is

$$V = \int_0^{2\pi}\int_0^{\pi} f\left(x \sin u \cos v + y \sin u \sin v + z \cos u + \frac{t}{k}, u, v\right) du\, dv,$$

where f is an arbitrary function of the three arguments

$$x \sin u \cos v + y \sin u \sin v + z \cos u + \frac{t}{k},\ u\ \textit{and}\ v.$$

§ 5.

Deductions from the general solution of § 4.

1°. *The analysis of wave-motions.* We shall now deduce from the general solution thus obtained a result relating to the analysis of those phenomena which are represented by solutions of the equation

$$\frac{\partial^2 V}{\partial x^2}+\frac{\partial^2 V}{\partial y^2}+\frac{\partial^2 V}{\partial z^2}=k^2\frac{\partial^2 V}{\partial t^2}.$$

If we revert to the fundamental idea of the definite integral as the limit of a sum of an infinite number of terms, we see that the general solution

$$V=\int_0^{2\pi}\int_0^{\pi} f\left(\sin u\cos v+y\sin u\sin v+z\cos u+\frac{t}{k},\ u,\ v\right)du\,dv$$

can be interpreted as meaning that V is the sum of an infinite number of terms of the type

$$f\left(x\sin u\cos v+y\sin u\sin v+z\cos u+\frac{t}{k},\ u,\ v\right),$$

there being one of these terms corresponding to every direction in space given by the direction-cosines

$$\sin u\cos v,\quad \sin u\sin v,\quad \cos u.$$

The solution V can therefore be regarded as the sum of constituent solutions, each of the type

$$F\left(x\sin u\cos v+y\sin u\sin v+z\cos u+\frac{t}{k}\right)$$

where the function F varies from one direction (u, v) to another.

Now let us fix our attention on one of these constituent solutions F. If for some range of values of the quantity

$$x\sin u\cos v+y\sin u\sin v+z\cos u+\frac{t}{k},$$

the function F is finite and continuous, we can for this range of values express F by Fourier's integral formula in the form

$$\frac{1}{\pi}\int_0^{\infty}d\lambda\int_a^b F(\alpha)\cos\left\{\lambda\left(x\sin u\cos v+y\sin u\sin v+z\cos u+\frac{t}{k}\right)-\lambda\alpha\right\}d\lambda\,d\alpha,$$

where a and b are the terminals of this range of values; or supposing the integration with respect to α to be performed,

$$\int_0^{\infty}g(\lambda)\,{\cos \atop \sin}\left\{\lambda\left(x\sin u\cos v+y\sin u\sin v+z\cos u+\frac{t}{k}\right)\right\}d\lambda,$$

where $g(\lambda)$ denotes some function of λ.

Now let us again revert to the idea of the definite integral as the limit of a sum. Then this latter integral can be regarded as the sum of an infinite number of terms of the type

$$\frac{\cos}{\sin}\left\{\lambda\left(x \sin u \cos v + y \sin u \sin v + z \cos u + \frac{t}{k}\right)\right\},$$

each term being multiplied by some factor depending on λ.

The solution V can therefore be regarded as constituted by the superposition of terms of this last type. But a term of this type represents a *simple uniform plane wave;* for on transforming the axes so that the new axis of x is the line whose direction-cosines are

$$\sin u \cos v, \quad \sin u \sin v, \quad \cos u,$$

the term becomes

$$\frac{\cos}{\sin}\lambda\left(x + \frac{t}{k}\right),$$

which represents a simple plane wave whose direction of propagation is the new axis of x. We see therefore that *the general finite solution of the differential equation of wave-motions,*

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = k^2 \frac{\partial^2 V}{\partial t^2}$$

can be analysed into simple plane waves, represented by terms of the type

$$F(\lambda, u, v)\frac{\cos}{\sin}\left\{\lambda\left(x \sin u \cos v + y \sin u \sin v + z \cos u + \frac{t}{k}\right)\right\}.$$

It is interesting to observe that Dr. Johnstone Stoney in 1897*) shewed by physical reasoning, and without any reference to the equation

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = k^2 \frac{\partial^2 V}{\partial t^2}$$

that all the disturbances of the luminiferous ether arising from sources of certain kinds can be resolved into trains of plane waves.

2°. *Solution of the equation*

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} + V = 0.$$

If a solution W of the equation

$$\frac{\partial^2 W}{\partial x^2} + \frac{\partial^2 W}{\partial y^2} + \frac{\partial^2 W}{\partial z^2} = \frac{\partial^2 W}{\partial t^2}$$

be of the form Ve^{it}, where V is a function of x, y, z only, which does not involve t, then V clearly satisfies the equation

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} + V = 0,$$

*) *Philosoph. Magazine*, (V) XLIII.

and therefore, on reference to the general solution of the wave-motion equation found in § 4, we see that *the general solution of the equation*

$$\frac{\partial^2 V}{\partial x^2}+\frac{\partial^2 V}{\partial y^2}+\frac{\partial^2 V}{\partial z^2}+V=0$$

is

$$V=\int_0^{2\pi}\int_0^{\pi} e^{i(x\sin u\cos v+y\sin u\sin v+z\cos u)} f(u,v)\,du\,dv.$$

3°. *Deduction of the known particular solutions of the equation*

$$\frac{\partial^2 V}{\partial x^2}+\frac{\partial^2 V}{\partial y^2}+\frac{\partial^2 V}{\partial z^2}+V=0.$$

It is known that particular solutions of the equation

$$\frac{\partial^2 V}{\partial x^2}+\frac{\partial^2 V}{\partial y^2}+\frac{\partial^2 V}{\partial z^2}+V=0$$

exist, which are of the form

$$V=r^{-\frac{1}{2}} J_{n+\frac{1}{2}}(r)\ P_n^m(\cos\theta)\ {\cos \atop \sin} m\varphi$$

$$(n=0,1,2,\cdots;\ m=0,1,2,\cdots,n),$$

where r, θ, φ are the polar coordinates corresponding to x, y, z. We shall now shew how these may be derived from the general solution of the equation which has just been found.

For let the general solution be written in the form

$$V=\int_0^{2\pi}\int_0^{\pi} e^{i(x\sin u\cos v+y\sin u\sin v+z\cos u)} f(u,v)\sin u\,du\,dv,$$

where $f(u, v)$ is an arbitrary function of the two arguments u and v, which may without loss of generality be taken to be periodic in u and v.

Now let the function $f(u, v)$ be expanded in surface-harmonics of u and v, so that

$$V=\sum_{n=0}^{\infty}\int_0^{2\pi}\int_0^{\pi} e^{i(x\sin u\cos v+y\sin u\sin v+z\cos u)}\, Y_n(u,v)\sin u\,du\,dv$$

where Y_n is a surface-harmonic of order n, i. e., if

$$\xi=\varrho\sin u\cos v,\quad \eta=\varrho\sin u\sin v,\quad \zeta=\varrho\cos u,$$

are regarded as the co-ordinates of a point in space, then $\varrho^n Y_n(u, v)$ is a homogeneous polynomial of degree n in ξ, η, ζ, satisfying Laplace's equation

$$\frac{\partial^2 V}{\partial \xi^2}+\frac{\partial^2 V}{\partial \eta^2}+\frac{\partial^2 V}{\partial \zeta^2}=0.$$

Next, let the variables be changed by the substitution

$$\cos u = \cos\theta\cos\omega + \sin\theta\sin\omega\cos v',$$
$$\sin u\sin(\varphi - v) = \sin\omega\sin v',$$
$$\sin u\cos(\varphi - v) = \cos\omega\sin\theta - \sin\omega\cos v'\cos\theta,$$

so that $(\varrho\sin\omega\cos v', \varrho\sin\omega\sin v', \varrho\cos\omega)$ are the co-ordinates of the point (ξ, η, ζ) referred to new axes, the line whose direction-cosines are $(\sin\theta\cos\varphi, \sin\theta\sin\varphi, \cos\theta)$ being taken as the new axis of z.

Thus

$$V = \sum_{n=0}^{\infty}\int_0^{2\pi}\int_0^{\pi} e^{ir\cos\omega}\, Y_n(u, v)\sin\omega\, d\omega\, dv'.$$

But a surface-harmonic of any order n remains a surface-harmonic of order n under any transformation of axes in which the origin is unchanged: and therefore $Y_n(u, v)$ is a surface harmonic of order n in ω and v'; and consequently it can be expanded in the form

$$A_n(\theta,\varphi)P_n(\cos\omega) + A_n^1(\theta,\varphi)P_n^1(\cos\omega)\cos v' + A_n^2(\theta,\varphi)P_n^2(\cos\omega)\cos 2v' + \cdots + A_n^n(\theta,\varphi)P_n^n(\cos\omega)\cos nv'$$
$$+ B_n'(\theta,\varphi)P_n'(\cos\omega)\sin v' + \cdots + B_n^n(\theta,\varphi)P_n^n(\cos\omega)\sin nv',$$

where $A_n(\theta,\varphi), \cdots, B_n^n(\theta,\varphi)$ are functions of θ and φ. Substituting this value for $Y_n(u, v)$ in the integral, and performing the integration with respect to v', we have

$$V = \sum_{n=0}^{\infty} A_n(\theta,\varphi)\int_0^{\pi} e^{ir\cos\omega} P_n(\cos\omega)\sin\omega\, d\omega;$$

and in virtue of the relation*)

$$\int_0^{\pi} e^{ir\cos\omega} P_n(\cos\omega)\sin\omega\, d\omega = \left(\frac{\pi}{2}\right)^{\frac{1}{2}} \frac{i^n J_{n+\frac{1}{2}}(r)}{\sqrt{r}},$$

this can be written in the form

$$V = \sum_{n=0}^{\infty} r^{-\frac{1}{2}} J_{n+\frac{1}{2}}(r) f_n(\theta,\varphi)$$

where $f_n(\theta,\varphi)$ denotes some function of θ and φ.

*) A proof of this and several related results will be found in a paper shortly to be published by the author.

Since the surface-harmonics $Y_n(\theta, \varphi)$ were independent of each other, the functions $f_n(\theta, \varphi)$, will be independent of each other and therefore each of the quantities

$$r^{-\frac{1}{2}} J_{n+\frac{1}{2}}(r) f_n(\theta, \varphi)$$

will be a solution of the equation

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} + V = 0.$$

But on transforming this equation to polar co-ordinates, and substituting the expression

$$r^{-\frac{1}{2}} J_{n+\frac{1}{2}}(r) f_n(\theta, \varphi)$$

for V, we find that the function $f_n(\theta, \varphi)$ must satisfy the differential equation for a surface-harmonic in θ and φ of order n. It follows that $f_n(\theta, \varphi)$ can be expanded in the form

$$f_n(\theta, \varphi) = A_n P_n(\cos\theta) + A_n^1 \cos\varphi P_n^1(\cos\theta) + \cdots + A_n \cos n\varphi P_n^n(\cos\theta)$$
$$+ B_n^1 \sin\varphi P_n^1(\cos\theta) + \cdots + B_n^n \sin n\varphi P_n^n(\cos\theta),$$

and thus *the particular solutions*

$$r^{-\frac{1}{2}} J_{n+\frac{1}{2}}(r) P_n^m(\cos\theta) \begin{matrix}\cos \\ \sin\end{matrix} m\varphi$$

are obtained.

Moreover, it is clear from the above proof that *in order to expand any solution*

$$V = \int_0^{2\pi}\int_0^{\pi} e^{i(x\sin u\cos v + y\sin u\sin v + z\cos u)} f(u, v) \sin u \, du \, dv$$

of the equation

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} + V = 0$$

as a series of the form

$$\sum_{n=0}^{\infty} r^{-\frac{1}{2}} J_{n+\frac{1}{2}}(r) Y_n(\theta, \varphi),$$

where Y_n is a surface-harmonic of order n in θ and φ, it is only necessary to expand the function $f(u, v)$ in surface-harmonics of u and v.

4°. *Expression of the solution of the equation*

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} + V = 0$$

as a series of generalised Bessel functions.

Another analysis of the solutions of the equation

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} + V = 0,$$

entirely different from that given in 3°, can be found in the following way.

Consider the expression

$$e^{\frac{1}{4}x\left(s-\frac{1}{s}\right)\left(t+\frac{1}{t}\right) - \frac{i}{4}y\left(s-\frac{1}{s}\right)\left(t-\frac{1}{t}\right) + \frac{i}{2}z\left(s+\frac{1}{s}\right)};$$

if this expression be regarded as a function of s and t, it can for finite non-zero values of s and t be expanded as a series of (positive and negative) integral powers of s and t, the coefficients in this series being functions of x, y and z. Let the coefficient of the term in $s^m t^n$ be denoted by $J_{m,n}(x, y, z)$: so that we have the relation

$$e^{\frac{1}{4}x\left(s-\frac{1}{s}\right)\left(t+\frac{1}{t}\right) - \frac{i}{4}y\left(s-\frac{1}{s}\right)\left(t-\frac{1}{t}\right) + \frac{i}{2}z\left(s+\frac{1}{s}\right)} = \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} J_{m,n}(x, y, z)\, s^m t^n.$$

This equation can be regarded as a generalisation of the equation

$$e^{\frac{1}{2}z\left(t-\frac{1}{t}\right)} = \sum_{n=-\infty}^{\infty} J_n(z)\, t^n,$$

which defines the ordinary Bessel functions; and we shall consequently call the functions $J_{m,n}(x, y, z)$ *generalised Bessel functions.*

We now proceed to establish some properties of the functions $J_{m,n}(x, y, z)$; it will be seen that they are very similar to those of the ordinary Bessel functions.

In the first place, since the expression

$$V = e^{\frac{1}{4}x\left(s-\frac{1}{s}\right)\left(t+\frac{1}{t}\right) - \frac{i}{4}y\left(s-\frac{1}{s}\right)\left(t-\frac{1}{t}\right) + \frac{i}{2}z\left(s+\frac{1}{s}\right)}$$

satisfies the equation

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} + V = 0,$$

it follows that *each of the functions* $J_{m,n}(x, y, z)$ *satisfies the equation*

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} + V = 0.$$

In the second place, we shall obtain an expression for $J_{m,n}(x, y, z)$ as a definite integral. By Laurent's theorem, we know that the coefficient of s^m in the expansion of

$$e^{\frac{1}{4}x\left(s-\frac{1}{s}\right)\left(t+\frac{1}{t}\right) - \frac{i}{4}y\left(s-\frac{1}{s}\right)\left(t-\frac{1}{t}\right) + \frac{i}{2}z\left(s+\frac{1}{s}\right)}$$

is

$$\frac{1}{2\pi i}\int_C s^{-m-1} e^{\frac{1}{4}x\left(s-\frac{1}{s}\right)\left(t+\frac{1}{t}\right)-\frac{i}{4}y\left(s-\frac{1}{s}\right)\left(t-\frac{1}{t}\right)+\frac{i}{2}z\left(s+\frac{1}{s}\right)}\,ds,$$

where C is any simple contour in the s-plane surrounding the origin; and again applying Laurent's theorem, the coefficient of t^n in this expression is seen to be

$$\frac{1}{4\pi^2}\int_C\int_D s^{-m-1}t^{-n-1} e^{\frac{1}{4}x\left(s-\frac{1}{s}\right)\left(t+\frac{1}{t}\right)-\frac{i}{4}y\left(s-\frac{1}{s}\right)\left(t+\frac{1}{t}\right)+\frac{i}{2}z\left(s+\frac{1}{s}\right)}\,ds\,dt,$$

where D is any simple contour in the t-plane surrounding the origin.

Now write $s = e^{iu}$, $t = e^{iv}$. Thus *we have the result*

$$J_{m,n}(x,y,z) = \frac{1}{4\pi^2}\int_0^{2\pi}\int_0^{2\pi} e^{-miu-niv+ix\sin u\cos v+iy\sin u\sin v+iz\cos u}\,du\,dv,$$

which may be regarded as the analogue of Bessel's integral

$$J_n(z) = \frac{1}{\pi}\int_0^{\pi}\cos(nu - z\sin u)\,du.$$

The functions $J_{m,n}(x,y,z)$ likewise possess an additiontheorem: for we have

$$e^{\frac{1}{4}(x+a)\left(s-\frac{1}{s}\right)\left(t+\frac{1}{t}\right)-\frac{i}{4}(y+b)\left(s-\frac{1}{s}\right)\left(t-\frac{1}{t}\right)+\frac{i}{2}(z+c)\left(s+\frac{1}{s}\right)}$$

$$= e^{\frac{1}{4}x\left(s-\frac{1}{s}\right)\left(t+\frac{1}{t}\right)-\frac{i}{4}y\left(s-\frac{1}{s}\right)\left(t-\frac{1}{t}\right)+\frac{i}{2}z\left(s+\frac{1}{s}\right)}$$

$$\times e^{\frac{1}{4}a\left(s-\frac{1}{s}\right)\left(t+\frac{1}{t}\right)-\frac{i}{4}b\left(s-\frac{1}{s}\right)\left(t-\frac{1}{t}\right)+\frac{i}{2}c\left(s+\frac{1}{s}\right)}$$

and so

$$\sum_{m,n} J_{m,n}(x+a,\ y+b,\ z+c)\,s^m t^n$$

$$= \sum_{m,n} J_{m,n}(x,y,z)\,s^m t^n \times \sum_{m,n} J_{m,n}(a,b,c)\,s^m t^n.$$

Equating coefficients on both sides of this equation, *we have the result*

$$J_{m,n}(x+a, y+b, z+c) = \sum_{p=-\infty}^{\infty}\sum_{q=-\infty}^{\infty} J_{p,q}(x,y,z)\,J_{m-p,n-q}(a,b,c),$$

which is the addition-theorem for the generalised Bessel functions, and is the analogue of the well-known result

$$J_n(z+c) = \sum_{p=-\infty}^{\infty} J_p(z)\,J_{n-p}(c).$$

We shall now shew how the generalised Bessel functions furnish an analysis of the general solution of the equation

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} + V = 0.$$

For the general solution is, by 2°,

$$V = \int_0^{\pi} \int_0^{2\pi} e^{i(x \sin u \cos v + y \sin u \sin v + z \cos u)} f(u, v)\, du\, dv,$$

where $f(u, v)$ can without loss of generality be taken to be a periodic function of u and v.

Now let the function $f(u, v)$ be expanded by the extended form of Fourier's theorem, in the form

$$f(u, v) = \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} a_{m,n} e^{imu + inv}.$$

Then we have

$$V = \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} a_{m,n} \int_0^{\pi} \int_0^{2\pi} e^{i(x \sin u \cos v + y \sin u \sin v + z \cos u + mu + nv)}\, du\, dv.$$

Comparing this with the form just found for the generalised Bessel functions, we see that *the general solution of the equation*

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} + V = 0$$

can be written

$$V = \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} a_{m,n} J_{m,n}(x, y, z),$$

where the quantities $a_{m,n}$ are arbitrary constants. This furnishes an alternative analysis of the solution to that given in 2°.

5°. *Gravitation and Electrostatic Attraction explained as modes of Wave-disturbance.*

The result of 1°, namely that any solution of the equation

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = k^2 \frac{\partial^2 V}{\partial t^2}$$

can be analysed into simple plane waves, throws a new light on the nature of those forces, such as gravitation and electrostatic attraction, which vary as the inverse square of the distance. For if a system of forces of this character be considered, their potential (or their component in any given direction) satisfies the equation

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = 0.$$

and therefore *à fortiori* it satisfies the equation

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = k^2 \frac{\partial^2 V}{\partial t^2}$$

where k is any constant. It follows from 1° that this potential (or force-component) can be analysed into simple plane waves in various directions, each wave being propagated with constant velocity. These waves interfere with each other in such a way that, when the action has once been set up, the disturbance at any point does not vary with the time, and depends only on the coordinates (x, y, z) of the point.

It is not difficult to construct, synthetically, systems of coexistent simple waves, having this property that the total disturbance at any point (due to the sum of all the waves) varies from point to point, but does not vary with the time. A simple example of such a system in the following.

Suppose that a particle is emitting spherical waves, such that the disturbance at a distance r from the origin, at time t, due to those waves whose wave-length lies between $\frac{2\pi}{\mu}$ and $\frac{2\pi}{\mu + d\mu}$, is represented by

$$\frac{2\,d\mu}{\pi\mu}\,\frac{\sin(\mu Vt - \mu r)}{r}$$

where V is the velocity of propagation of the waves. Then after the waves have reached the point r, so that $(Vt - r)$ is positive, the total disturbance at this point (due to the sum of all the waves) is

$$\int_0^\infty \frac{2\,d\mu}{\pi\mu}\,\frac{\sin(\mu Vt - \mu r)}{r}.$$

Take $\mu Vt - \mu r = y$, where y is a new variable. Then this disturbance is

$$\frac{2}{\pi r}\int_0^\infty \frac{\sin y}{y}\,dy;$$

or, since

$$\int_0^\infty \frac{\sin y}{y}\,dy = \frac{\pi}{2},$$

it is

$$\frac{1}{r}.$$

The total disturbance at any point, due to this system of waves, is therefore independent of the time, and is everywhere proportional to the gravitational potential due to the particle at the point.

On an Expression of an Electromagnetic Field Due to Electrons By Means of Two Scalar Potential Functions (back)

On an Expression of the Electromagnetic Field Due to

Electrons by Means of Two Scalar Potential Functions

Proceedings of the London Mathematical Society

ON AN EXPRESSION OF THE ELECTROMAGNETIC FIELD DUE TO ELECTRONS BY MEANS OF TWO SCALAR POTENTIAL FUNCTIONS

By E. T. WHITTAKER.

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1. *Object of Paper.*

The object of the present paper is to show that when any number of electrons are moving in any manner the functions which define the resulting electrodynamic field, namely, the three components of dielectric displacement in the æther and the three components of the magnetic force at every point of the field, can be expressed in terms of the derivates of two scalar potential functions. (Previous writers have expressed them in terms of a scalar potential function and a vector potential function, which are equivalent to *four* scalar potential functions.) These two scalar potential functions are explicitly evaluated in terms of the charges and co-ordinates of the electrons. It is then shown that from these results the general functional form of an electrodynamic disturbance due to electrons can be derived.

2. *Explanation of Notation, and Summary of previously known Results.*

The work of previous writers, so far as it concerns the present investigation and explains the notation used, may be briefly summarized as follows:—

Let ρ be the volume density of electricity at any place and time, and let v_x, v_y, v_z be the components of its velocity, and c the velocity of light in the æther. Let d_x, d_y, d_z be the three components of the dielectric displacement in the æther, and h_x, h_y, h_z the three components of the magnetic force. Then the fundamental equations of electrodynamics may be written in Lorentz's form (the units being suitably chosen):

$$\frac{\partial \rho}{\partial t}+\frac{\partial(\rho v_x)}{\partial x}+\frac{\partial(\rho v_y)}{\partial y}+\frac{\partial(\rho v_z)}{\partial z}=0,$$

$$\frac{\partial d_x}{\partial x}+\frac{\partial d_y}{\partial y}+\frac{\partial d_z}{\partial z}=\rho, \qquad \frac{\partial h_x}{\partial x}+\frac{\partial h_y}{\partial y}+\frac{\partial h_z}{\partial z}=0,$$

$$c\left(\frac{\partial h_z}{\partial y}-\frac{\partial h_y}{\partial z}\right)=\frac{\partial d_x}{\partial t}+\rho v_x, \qquad c\left(\frac{\partial d_z}{\partial y}-\frac{\partial d_y}{\partial z}\right)=-\frac{\partial h_x}{\partial t},$$

$$c\left(\frac{\partial h_x}{\partial z}-\frac{\partial h_z}{\partial x}\right)=\frac{\partial d_y}{\partial t}+\rho v_y, \qquad c\left(\frac{\partial d_x}{\partial z}-\frac{\partial d_z}{\partial x}\right)=-\frac{\partial h_y}{\partial t},$$

$$c\left(\frac{\partial h_y}{\partial x}-\frac{\partial h_x}{\partial y}\right)=\frac{\partial d_z}{\partial t}+\rho v_z, \qquad c\left(\frac{\partial d_y}{\partial x}-\frac{\partial d_x}{\partial y}\right)=-\frac{\partial h_z}{\partial t}.$$

In place of d_x, d_y, d_z, h_x, h_y, h_z, we can define the field by a scalar potential function ϕ and three functions a_x, a_y, a_z, which are usually regarded as the three components of a vector potential. The quantities d_x, d_y, d_z, h_x, h_y, h_z are given in terms of ϕ, a_x, a_y, a_z by the equations

$$d_x=-\frac{1}{c}\frac{\partial a_x}{\partial t}-\frac{\partial\phi}{\partial x}, \qquad h_x=\frac{\partial a_z}{\partial y}-\frac{\partial a_y}{\partial z},$$

and four similar equations for d_y, d_z, h_y, h_z.

The scalar potential and the three components of the vector potential satisfy the system of equations

$$c^2\nabla^2\phi-\frac{\partial^2\phi}{\partial t^2}=-c^2\rho, \qquad c^2\nabla^2 a_x-\frac{\partial^2 a_x}{\partial t^2}=-c\rho v_x,$$

$$c^2\nabla a_y-\frac{\partial^2 a_y}{\partial t^2}=-c\rho v_y, \qquad c^2\nabla^2 a_z-\frac{\partial^2 a_z}{\partial t^2}=-c\rho v_z,$$

$$\frac{\partial a_x}{\partial x}+\frac{\partial a_y}{\partial y}+\frac{\partial a_z}{\partial z}+\frac{1}{c}\frac{\partial\phi}{\partial t}=0.$$

For the fundamental case, namely, that in which the field is due to any number of electrons moving in any way, the scalar potential and the three components of the vector potential are given by the equations

$$\phi(x, y, z, t)=\Sigma\frac{ec}{4\pi}\frac{1}{c\bar{r}+\bar{r}\bar{v}\cos(\bar{v},\bar{r})}, \quad a_x(x, y, z, t)=\Sigma\frac{e}{4\pi}\frac{\bar{v}_x}{c\bar{r}+\bar{r}\bar{v}\cos(\bar{v},\bar{r})},$$

and two similar equations for a_y and a_z, where e is the charge on a typical electron, r is its distance from the point (x, y, z), v is its velocity, (v_x, v_y, v_z) the components of v, (v, r) the angle between the direction of v and r, and the bars over the letters mean that the position of the electron considered is that which it occupied at a time $t-\bar{r}/c$; and the summation is taken over all the electrons. We shall assume throughout the paper that the velocities of all the electrons are less than the velocity of radiation.

3. *Introduction and Evaluation of the two Scalar Potentials.*

Now let $x'(t)$, $y'(t)$, $z'(t)$ denote the position of the electron e at time t; and let $\bar{x}'$ be used to denote $x'(t-\bar{r}/c)$, so that $\bar{x}'$, $\bar{y}'$, $\bar{z}'$ are known

The question now naturally suggests itself, whether the most general solution of Laplace's equation can be represented by an expression of this type. We shall shew that the answer to this is in the affirmative.

For let $V(x, y, z)$ be any solution (single-valued or many-valued) of the equation

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = 0.$$

Let (x_0, y_0, z_0) be some point at which some branch of the function $V(x, y, z)$ is regular. Then if we write

$$x = x_0 + X, \quad y = y_0 + Y, \quad z = z_0 + Z$$

it follows that for all points situated within a finite domain surrounding the point (x_0, y_0, z_0), this branch of the function $V(x, y, z)$ can be expanded in an absolutely and uniformly convergent series of the form

$$V = a_0 + a_1 X + b_1 Y + c_1 Z + a_2 X^2 + b_2 Y^2 + c_2 Z^2 + d_2 YZ \\ + e_2 ZX + f_2 XY + a_3 X^3 + \cdots.$$

Substituting this expansion in Laplace's equation, which can be written

$$\frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2} + \frac{\partial^2 V}{\partial Z^2} = 0,$$

and equating to zero the coefficients of the various powers of X, Y and Z, we obtain an infinite number of linear relations, namely

$$a_2 + b_2 + c_2 = 0, \text{ etc.}$$

between the constants in the expansion.

There are $\frac{1}{2}n(n-1)$ of these relations between the $\frac{1}{2}(n+1)(n+2)$ coefficients of the terms of any degree n in the expansion of V; so that only $\left\{\frac{1}{2}(n+1)(n+2) - \frac{1}{2}n(n-1)\right\}$ or $(2n+1)$ of the coefficients of terms of degree n in the expansion of V are really independent. It follows that the terms of degree n in V must be a linear combination of $(2n+1)$ linearly independent particular solutions of Laplace's equation, which are of degree n in X, Y, Z.

To find these solutions, consider the expansion of the quantity

$$(Z + iX\cos u + iY\sin u)^n$$

as a sum of sines and cosines of multiples of u, in the form

$$(Z + iX\cos u + iY\sin u)^n = g_0(X, Y, Z) + g_1(X, Y, Z)\cos u \\ + g_2(X, Y, Z)\cos 2u + \cdots + g_n(X, Y, Z)\cos nu \\ + h_1(X, Y, Z)\sin u + h_2(X, Y, Z)\sin 2u + \cdots \\ + h_n(X, Y, Z)\sin nu.$$

Combining these results with the expressions already found for ϕ, a_x, a_y, a_z, we have

$$\frac{\partial\psi}{\partial x} - \frac{\partial G}{\partial y} = a_x, \qquad \frac{\partial\psi}{\partial y} + \frac{\partial G}{\partial x} = a_y, \qquad \frac{\partial\psi}{\partial z} + \frac{1}{c}\,\frac{\partial F}{\partial t} = a_z,$$

$$\frac{1}{c}\,\frac{\partial\psi}{\partial t} + \frac{\partial F}{\partial z} = -\phi.$$

Substituting these results for ϕ, a_x, a_y, a_z in the equations of the type

$$d_x = -\frac{1}{c}\,\frac{\partial a_x}{\partial t} - \frac{\partial\phi}{\partial x}, \qquad h_x = \frac{\partial a_z}{\partial y} - \frac{\partial a_y}{\partial z},$$

which give the components of the dielectric displacement and the magnetic force, *we find that ψ disappears automatically*, and we obtain

$$d_x = \frac{\partial^2 F}{\partial x\,\partial z} + \frac{1}{c}\,\frac{\partial^2 G}{\partial y\,\partial t}, \qquad d_y = \frac{\partial^2 F}{\partial y\,\partial z} - \frac{1}{c}\,\frac{\partial^2 G}{\partial x\,\partial t}, \qquad d_z = \frac{\partial^2 F}{\partial z^2} - \frac{1}{c^2}\,\frac{\partial^2 F}{\partial t^2};$$

$$h_x = \frac{1}{c}\,\frac{\partial^2 F}{\partial y\,\partial t} - \frac{\partial^2 G}{\partial x\,\partial z}, \qquad h_y = -\frac{1}{c}\,\frac{\partial^2 F}{\partial x\,\partial t} - \frac{\partial^2 G}{\partial y\,\partial z}, \qquad h_z = \frac{\partial^2 G}{\partial x^2} + \frac{\partial^2 G}{\partial y^2}.$$

These equations show that the six components of the dielectric displacement and the magnetic force can be expressed in terms of the derivates of two scalar potentials F and G, defined by the equations

$$F(x, y, z, t) = \Sigma\,\frac{e}{4\pi}\,\sinh^{-1}\frac{\bar{z}' - z}{\{(\bar{x}' - x)^2 + (\bar{y}' - y)^2\}^{\frac{1}{2}}},$$

$$G(x, y, z, t) = \Sigma\,\frac{e}{4\pi}\,\tan^{-1}\frac{\bar{y}' - y}{\bar{x}' - x},$$

where the summation is taken over all the electrons in the field.

It can without difficulty be shown that, if any number of electrons whose total charge is zero are moving in any manner so as to remain always in the vicinity of a given point (*i.e.*, to be in *stationary* motion), then the electromagnetic field thus generated is of the type given by

$$F = \frac{1}{r}\,f\left(t - \frac{r}{c}\right), \qquad G = 0,$$

where r is the distance from the point and f is an arbitrary function; or, more generally, of a field of this type superposed on fields of the same type, but related to the axes of y and x in the same way as this is related to the axis of z. This is perhaps of some interest in connection with the view advocated by some physicists that the atoms of the chemical elements consist of sets of electrons, whose total charge is zero, in stationary motion.

4. *Discussion of the Apparent Asymmetry of the preceding Result, and its Vector Expression.*

The formulæ thus obtained are not symmetrical with respect to x, y, and z. In order to discuss their relation to symmetrical formulæ, we observe that they can be written in the form of vector equations

$$d = \text{curl curl } f + \text{curl } \frac{1}{c} \dot{g}, \qquad h = \text{curl } \frac{1}{c} \dot{f} - \text{curl curl } g,$$

where d and h are the electric and magnetic vectors, and f and g are vectors directed parallel to the axis of z, whose magnitudes are F and G respectively. These vector equations are quite symmetrical, and our result is that, if, instead of regarding the electromagnetic field as defined by the vectors d and h, we regard it as defined by vectors f and g, connected with d and h by the above vector equations, then f and g are simple functions of the coordinates of the electrons, whereas d and h are complicated functions of their velocities and accelerations; and we have also obtained the result that without loss of generality we can take f and g to be everywhere, and at all times, parallel to some fixed direction in space (*e.g.*, the axis of z), a fact which makes it possible to specify them by two scalar quantities only.

It might be asked whether vectors f and g exist which satisfy the above vector equations and which are perfectly symmetrical—the answer to this is in the negative; in fact, although the equations are themselves invariantive, and can therefore be expressed in the vector notation, yet they do not possess invariant solutions; just as the vector equation

$$\text{grad}\left(\frac{1}{r}\right) = \text{curl } a$$

(where r is the scalar distance from the origin and a is a vector to be determined) possesses an infinite number of solutions a, which can readily be found, but each of which is specially related to some line in space, so that no solution is symmetrical.

5. *Deduction of the General Functional Form of an Electrodynamic Disturbance in the Æther.*

Having now shown that an electrodynamic field due to electrons is completely characterized by two scalar potential functions F and G, we can proceed to deduce its general functional form.

The functions F and G have singularities at those points which are actually occupied by electrons; at all other points we find by direct

2 B 2

Planetary Position Effect on Short-Wave Signal Quality (back)

Planetary Position Effect on Short-Wave Signal Quality

J. H. NELSON

Planetary Position Effect on Short-Wave Signal Quality

J. H. NELSON

A new approach to an as yet unsolved problem is the observance of planetary effects on transatlantic short-wave radio signals. Correlation over seven years shows that certain planetary arrangements agree well with the behavior of short-wave signals.

AT THE Central Radio Office of RCA Communications, Inc., in lower Manhattan, an observatory housing a 6-inch refracting telescope is maintained for the observation of sunspots. The purpose of erecting this observatory in 1946 was to develop a method of forecasting radio storms from the study of sunspots. After about one year of experimenting, a forecasting system of short-wave conditions was inaugurated based upon the age, position, classification, and activity of sunspots. Satisfactory results were obtained, but failure of this system from time to time, indicated that phenomena other than sunspots needed to be studied. The first article[1] by the author on this subject appeared in March 1951; the current article is in part a review of that article, and in part will submit additional evidence supporting deductions made at that time.

STUDY OF PLANET POSITIONS

CYCLIC variations in sunspot activity have been studied by many solar investigators in the past and attempts were made by some, notably Huntington,[2] Clayton,[3] and Sanford,[4] to connect these variations to planetary influences. The books of these three investigators were studied and their results found sufficiently encouraging to warrant correlating similar planetary interrelationships with radio signal behavior. However, it was decided to investigate the effects of all the planets from Mercury to Saturn* instead of only the major planets as they had done. The same heliocentric angular relationships of 0, 90, 180, and 270 degrees were used and dates when any two or more planets were separated by one of these angles were recorded.

Investigation quickly showed there was positive correlation between these planetary angles and transatlantic short-wave signal variations. Radio signals showed a tendency to become degraded within a day or two of planetary configurations of the type being studied. However, all configurations did not correspond to signal degradation. Certain configurations showed better correlation than others.

Considerable study was devoted to the most severe degradations and led to the discovery that when three planets held a "multiple of 90 degrees" arrangement among themselves, the correlation was more pronounced. These arrangements were called "multiple configurations" and exist when two planets are at 0 degree with each other and a third planet is 90 degrees or 180 degrees away from them. Also, a multiple exists when two planets are separated by 180 degrees with a third planet 90 degrees from each. These multiples are quite common. A more uncommon type of multiple is the case where all three planets are at 0 degree with each other. From the few cases recorded, this type of multiple shows the least correlation.

Many of the multiples are completed in the space of a few hours, being accompanied by sharp severe signal degradation. At other times, the multiple may take several days to pass, being accompanied by generally erratic conditions during the period. The time needed to complete the multiple depends on the relative speeds between the three or more planets involved. These multiples show correlation for plus and minus about 5 degrees from the exact arrangements previously mentioned.

Configurations of this type actually can be considered as cycles and when several cycles peak at the same time there should be maximum effects. The records for 1948, 1949, and 1950 indicate that such has been the result. Specific instances are demonstrated in Cases 1 to 9 in Table I. Since consistency of data is of paramount importance in an article of this type, the same cycles between the same three planets have been selected. We may refer to these arrangements as multicycles.

All the close multicycles made between Mercury-Venus-Jupiter were extracted from the records of 1948, 1949, 1950

Table I. Multicycles Among Planets Affecting Radio Signal

Cases	Dates	Time Consumed	Results in Signal Degradation
1	Feb 23/48	1 day	Severe 23d and 24th
2	Apr 18–22/48	5 days	Severe 19th to 22d
3	June 19–23/48	5 days	Slight 19th to 22d
4	Aug 18–21/48	4 days	Slight 19th to 21st
5	Oct 15/48	1 day	Very severe 14th and 15th
6	Apr 12/49	1 day	Very severe 11th to 13th
7	Oct 6–8/49	3 days	Very severe 7th and 8th
8	Apr 2–5/50	4 days	Very severe 1st to 6th
9	Sept 28–Oct 1/50	4 days	Very severe 30th to 4th*
10	Sept 21–23/51	3 days	Extremely severe 20th to 26th**

* A multicycle between Mercury-Earth-Mars came on 5th and 6th. The degradation continued through to October 7.

** Three complete multicycles took place during this period involving Mercury, Venus, Jupiter, Saturn, and Uranus.

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* J. H. Clark of Press Wireless Inc., New York, N. Y., has found that Uranus, Neptune, and Pluto also show correlation. Mr. Clark has been correlating planetary positions and short-wave signal behavior using the methods given in the author's original article.

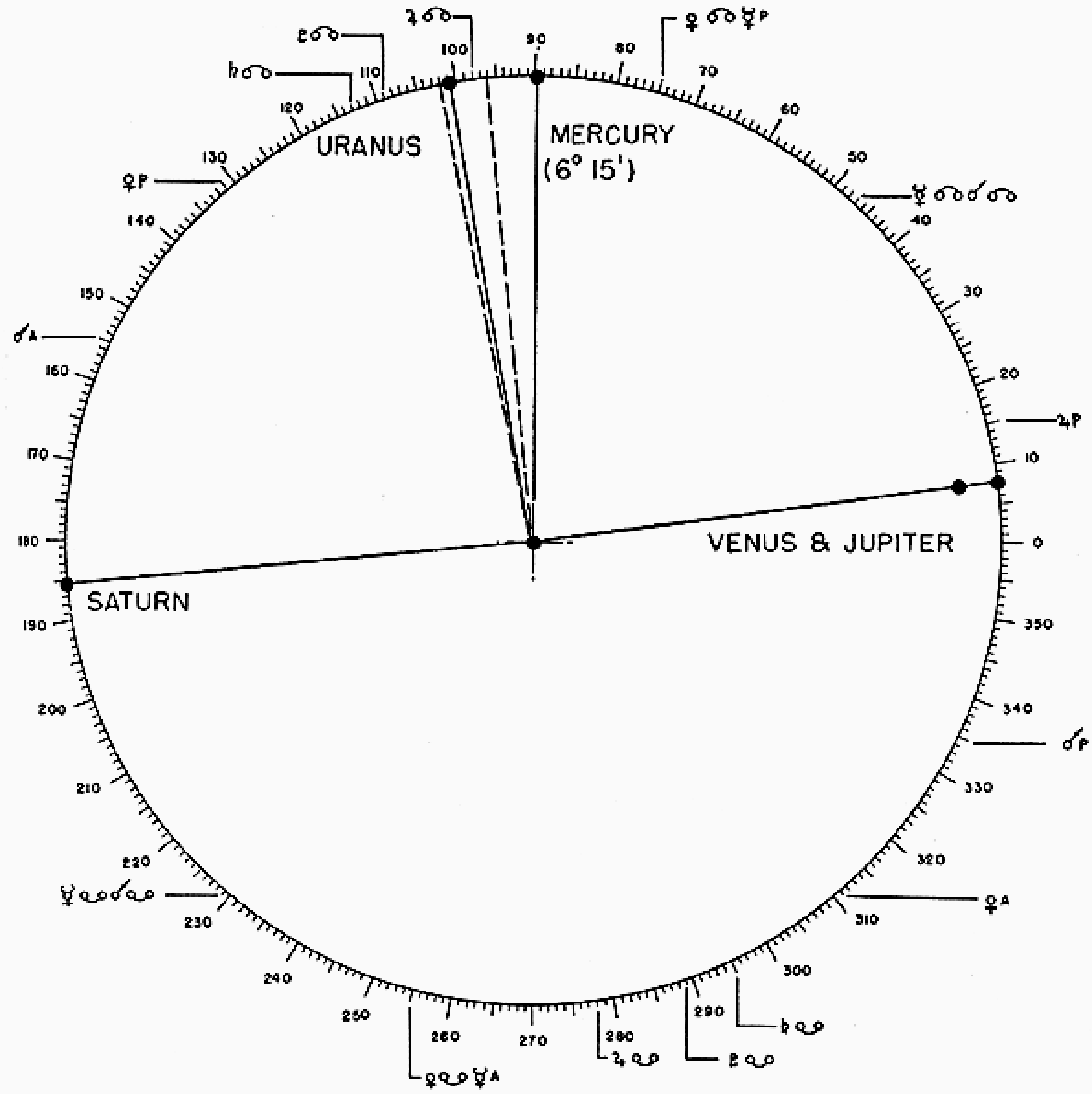

Figure 2. The heliocentric arrangement of the five planets shown which resulted in extremely severe signal degradation from September 20 to 26, 1951

influence on the sun for prolonged periods of time and therefore establish an over-all standard of disturbed or quiet conditions. However, the arrangements of the other slow planets can add to or take away from their effectiveness to some extent. Therefore, when Jupiter and Saturn are spaced near any multiple of 90 degrees, we should find the most degraded years with a high percentage of the radio disturbances severe.

The year 1951, which was a very degraded year, is an example of this. A slow planet multiple existed between Jupiter, Saturn, and Uranus with Jupiter and Saturn at nearly 180 degrees and Uranus almost 90 degrees from each. This arrangement set a low standard for 1951 and even normally weak cycles between isolated planets showed an effect. The radio disturbances were prolonged and severe. This multiple between these three slow planets permitted a great increase in fast planet multiples and semimultiples. A semimultiple took place each time a fast planet cycled with Jupiter, Saturn, and Uranus successively within a few days. This happened frequently in 1951.

The records indicate that when Jupiter and Saturn were spaced by a multiple of 60 degrees, radio signals were of better quality than when spaced by multiples of 90 degrees. Under such an arrangement there are fewer multicycles. During such years a high percentage of the single cycles show no important correlation except on the normally weaker circuits. Only the stronger groups of cycles are then accompanied by significant degradation.

It is worthy of note that in 1948 when Jupiter and Saturn were spaced by 120 degrees, and solar activity was at a maximum, radio signals averaged of far higher quality for the year than in 1951 with Jupiter and Saturn at 180 degrees and a considerable decline in solar activity. In other words, the average quality curve of radio signals followed the cycle curve between Jupiter and Saturn rather than the sunspot curve.

When all nine planets of the solar system are considered, we find a great many multicycles and individual cycles which would make correlation very difficult if these cycles were evenly spaced in time. However, the cycles are not evenly spaced, there being a general tendency for the cycles to occur in groups. There are, however, exceptions to this at times.

CONCLUSION

THE AUTHOR chooses to look upon this hypothesis of a planetary-positions effect upon the quality of short-wave signals as a new approach to the problem and it should be considered as one more tool with which a researcher might work. A tremendous amount of work needs yet to be done.

The correlation found between signal degradations and these planetary arrangements in the past has been sufficiently consistent to indicate that under these arrangements, particularly in the case of multicycles, the planets possibly influence the sun in such a manner as to cause a temporary change in its radiation characteristics. The ionosphere of the earth is apparently particularly sensitive to these changes and reacts accordingly.

By combining planetary indications with solar observations and a day-to-day signal analysis, a 24-hour forecasting system has been developed which averaged close to 85 per cent accuracy throughout 1950 and 1951 as reported by RCA Communications at Riverhead, L. I., N. Y.

REFERENCES

1. Shortwave Radio Propagation Correlation With Planetary Positions, J. H. Nelson. *RCA Review* (Princeton, N. J.), March 1951.

2. Earth and Sun (book), E. Huntington. Yale University Press, New Haven, Conn., 1923.

3. Solar Relations (book), H. Clayton. Clayton Weather Service, Canton, Mass., 1943.

4. Influence of Planetary Configurations Upon the Frequency of Visible Sunspots (book), F. Sanford. Smithsonian Institution, Washington, D. C., 1936.

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